AP Calculus

Introduction

An AP course in calculus consists of a full high school academic year of work that is comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both, from institutions of higher learning.

The AP Program includes specifications for two calculus courses and the exam for each course. The two courses and the two corresponding exams are designated as Calculus AB and Calculus BC.

Calculus AB can be offered as an AP course by any school that can organize a curriculum for students with mathematical ability. This curriculum should include all the prerequisites for a year's course in calculus listed on page 6. Calculus AB is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still cover the Calculus AB curriculum within a year. However, if students are to be adequately prepared for the Calculus AB Exam, most of the year must be devoted to the topics in differential and integral calculus described on pages 6 to 9. These topics are the focus of the AP Exam questions.

Calculus BC can be offered by schools that are able to complete all the prerequisites listed on page 6 before the course. Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics covered in Calculus AB plus additional topics, but both courses are intended to be challenging and demanding; they require a similar depth of understanding of common topics. The topics for Calculus BC are described on pages 9 to 13. A Calculus AB subscore grade is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics.

Both courses described here represent college-level mathematics for which most colleges grant advanced placement and/or credit. Most colleges and universities offer a sequence of several courses in calculus, and entering students are placed within this sequence according to the extent of their preparation, as measured by the results of an AP Exam or other criteria. Appropriate credit and placement are granted by each institution in accordance with local policies. The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB. Many colleges provide statements regarding their AP policies in their catalogs and on their Web sites.

Secondary schools have a choice of several possible actions regarding AP Calculus. The option that is most appropriate for a particular school depends on local conditions and resources: school size, curriculum, the preparation of teachers, and the interest of students, teachers, and administrators.

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses covering the equivalent of four full years of high school mathematics before attempting calculus. These courses include algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. Even though schools may choose from a variety of ways to accomplish these studies—including beginning the study of high school mathematics in grade 8; encouraging the election of more than one mathematics course in grade 9, 10, or 11; or instituting a program of summer study or guided independent study—it should be emphasized that eliminating preparatory course work in order to take an AP course is not appropriate.

The AP Calculus Development Committee recommends that calculus should be taught as a college-level course. With a solid foundation in courses taken before AP, students will be prepared to handle the rigor of a course at this level. Students who take an AP Calculus course should do so with the intention of placing out of a comparable college calculus course. This may be done through the AP Exam, a college placement exam, or any other method employed by the college.

The Courses

Philosophy

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of the numbers $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.

Topic Outline for Calculus AB*

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See AP Central [apcentral.collegeboard.com] and the *AP Calculus Teacher's Guide* for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity

***There are no major changes to the Topic Outline from the May 2004, May 2005 edition of this Course Description.**

- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric consequences
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables (including change of limits for definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Topic Outline for Calculus BC**

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central and the *AP Calculus Teacher's Guide* for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between

****There are no major changes to the Topic Outline from the May 2004, May 2005 edition of this Course Description.**

the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

* **Parametric, polar, and vector functions** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric consequences
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity

\+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration

- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

\+ Numerical solution of differential equations using Euler's method

\+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation

\+ Derivatives of parametric, polar, and vector functions

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

* **Applications of integrals** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric form).

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- \+ Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)
- \+ Improper integrals (as limits of definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line

- Solving separable differential equations and using them in modeling (in particular, studying the equation $y' = ky$ and exponential growth)

+ Solving logistic differential equations and using them in modeling

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

*IV. Polynomial Approximations and Series

* **Concept of series** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

* **Series of constants**

+ Motivating examples, including decimal expansion
+ Geometric series with applications
+ The harmonic series
+ Alternating series with error bound
+ Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
+ The ratio test for convergence and divergence
+ Comparing series to test for convergence or divergence

* **Taylor series**

+ Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
+ Maclaurin series and the general Taylor series centered at $x = a$
+ Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\dfrac{1}{1-x}$
+ Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
+ Functions defined by power series
+ Radius and interval of convergence of power series
+ Lagrange error bound for Taylor polynomials

Use of Graphing Calculators

Professional mathematics organizations such as the National Council of Teachers of Mathematics, the Mathematical Association of America, and the Mathematical Sciences Education Board of the National Academy of Sciences have strongly endorsed the use of calculators in mathematics instruction and testing.

The use of a graphing calculator in AP Calculus is considered an integral part of the course. Students should be using this technology on a regular basis so that they become adept at using their graphing calculators. Students should also have experience with the basic paper-and-pencil techniques of calculus and be able to apply them when technological tools are unavailable or inappropriate.

The AP Calculus Development Committee understands that new calculators and computers, capable of enhancing the teaching of calculus, continue to be developed. There are two main concerns that the committee considers when deciding what level of technology should be required for the exams: equity issues and teacher development.

Over time, the range of capabilities of graphing calculators has increased significantly. Some calculators are much more powerful than first-generation graphing calculators and may include symbolic algebra features. Other graphing calculators are, by design, intended for students studying mathematics at lower levels than calculus. The committee can develop exams that are appropriate for any given level of technology, but it cannot develop exams that are fair to all students if the spread in the capabilities of the technology is too wide. Therefore, the committee has found it necessary to make certain requirements of the technology that will help ensure that all students have sufficient computational tools for the AP Calculus Exams. Exam restrictions should not be interpreted as restrictions on classroom activities. The committee will continue to monitor the developments of technology and will reassess the testing policy regularly.

Graphing Calculator Capabilities for the Exams

The committee develops exams based on the assumption that all students have access to four basic calculator capabilities used extensively in calculus. A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

1) plot the graph of a function within an arbitrary viewing window,
2) find the zeros of functions (solve equations numerically),
3) numerically calculate the derivative of a function, and
4) numerically calculate the value of a definite integral.

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any exam question that requires the use of a calculator. Care is taken to ensure that the exam questions do not favor students who use graphing calculators with more extensive built-in features.

Students are expected to bring a calculator with the capabilities listed above to the exams. AP teachers should check their own students' calculators to ensure that the required conditions are met. A list of acceptable calculators can be found at AP Central. If a student wishes to use a calculator that is not on the list, the teacher must contact the AP Program (609 771-7300) before April 1 of the testing year to request written permission for the student to use the calculator on AP Exams.

Technology Restrictions on the Exams

Nongraphing scientific calculators, computers, devices with a QWERTY keyboard, and pen-input/stylus-driven devices, or electronic writing pads are not permitted for use on the AP Calculus Exams.

Test administrators are required to check calculators before the exam. Therefore, it is important for each student to have an approved calculator. The student should be thoroughly familiar with the operation of the calculator he or she plans to use. Calculators may not be shared, and communication between calculators is prohibited during the exam. Students may bring to the exam one or two (but no more than two) graphing calculators from the approved list.

Calculator memories will not be cleared. Students are allowed to bring calculators containing whatever programs they want.

Students must not use calculator memories to take test materials out of the room. Students should be warned that their grades will be invalidated if they attempt to remove test materials by any method.

Showing Work on the Free-Response Sections

Students are expected to show enough of their work for AP Exam Readers to follow their line of reasoning. To obtain full credit for the solution to a free-response problem, students must communicate their methods and conclusions clearly. Answers should show enough work so that the reasoning process can be followed throughout the solution. This is particularly important for assessing partial credit. Students may also be asked to use complete sentences to explain their methods or the reasonableness of their answers, or to interpret their results.

For results obtained using one of the four required calculator capabilities listed on page 14, students are required to write the setup (e.g., the equation being solved, or the derivative or definite integral being evaluated) that leads to the solution, along with the result produced by the calculator. For example, if the student is asked to find the area of a region, the student is expected to show a definite integral (i.e., the setup) and the answer. The student need not compute the antiderivative; the calculator may be used to calculate the value of the definite integral without further explanation. For solutions obtained using a calculator capability other than one of the four required ones, students must also show the mathematical steps necessary to produce their results; a calculator result alone is not sufficient. For example, if the student is asked to find a relative minimum value of a function, the student is expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a built-in minimum finder.

When a student is asked to justify an answer, the justification must include mathematical (noncalculator) reasons, not merely calculator results. Functions, graphs, tables, or other objects that are used in a justification should be clearly labeled.

A graphing calculator is a powerful tool for exploration, but students must be cautioned that exploration is not a mathematical solution. Exploration with a graphing calculator can lead a student toward an analytical solution, and after a solution is found, a graphing calculator can often be used to check the reasonableness of the solution.

As on previous AP Exams, if a calculation is given as a decimal approximation, it should be correct to three places after the decimal point unless otherwise indicated. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology; for example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

For more information on the instructions for the free-response sections, read the "Calculus FRQ Instruction Commentary" written by the AP Calculus Development Committee and the Chief Reader. It is available on the home pages for Calculus AB and Calculus BC at AP Central.

Beginning with the 2005 exams, sign charts by themselves will not be accepted as a sufficient response when a free-response question requires a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more detailed information on this policy change, read the article "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute Extrema" written by the AP Calculus Development Committee chair and the Chief Reader. It is available on the home pages for Calculus AB and Calculus BC at AP Central.

The Exams

The Calculus AB and BC Exams seek to assess how well a student has mastered the concepts and techniques of the subject matter of the corresponding courses. Each exam consists of two sections, as described below.

Section I: a multiple-choice section testing proficiency in a wide variety of topics

Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning

The time allotted for each AP Calculus Exam is 3 hours and 15 minutes. The multiple-choice section of each exam consists of 45 questions in 105 minutes. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

The free-response section of each exam has two parts: one part requiring graphing calculators and a second part not allowing graphing calculators. The AP Exams are designed to accurately assess student mastery of both the concepts and techniques of calculus. The two-part format for the free-response section provides greater flexibility in the types of problems that can be given while ensuring fairness to all students taking the exam, regardless of the graphing calculator used.

The free-response section of each exam consists of 6 problems in 90 minutes. Part A of the free-response section (3 problems in 45 minutes) contains some problems or parts of problems for which a graphing calculator is required. Part B of the free-response section (3 problems in 45 minutes) does not allow the use of a calculator. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time.

In determining the grade for each exam, the scores for Section I and Section II are given equal weight. Since the exams are designed for full coverage of the subject matter, it is not expected that all students will be able to answer all the questions.

Calculus AB Subscore Grade for the Calculus BC Exam

A Calculus AB subscore grade is reported based on performance on the portion of the exam devoted to Calculus AB topics (approximately 60% of the exam). The Calculus AB subscore grade is designed to give colleges and universities more information about the student. Although each college and university sets its own policy for awarding credit and/or placement for AP Exam grades, it is recommended that institutions apply the same policy to the Calculus AB subscore grade that they apply to the Calculus AB grade. Use of the subscore grade in this manner is consistent with the philosophy of the courses, since common topics are tested at the same conceptual level in both Calculus AB and Calculus BC.

The Calculus AB subscore grade was first reported for the 1998 Calculus BC Exam. The reliability of the Calculus AB subscore grade is nearly equal to the reliabilities of the AP Calculus AB and Calculus BC Exams.

Calculus AB: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus AB are included in the following sections. Answers to the sample questions are given on page 31.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

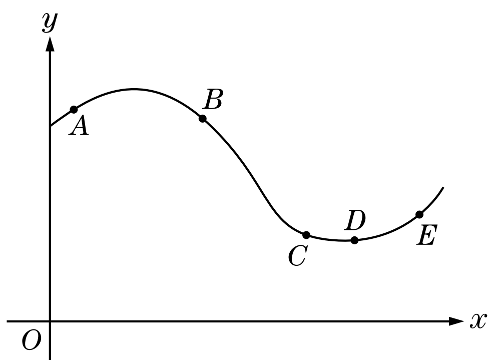
Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:
(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. What is $\lim\limits_{h \to 0} \dfrac{\cos\left(\dfrac{3\pi}{2} + h\right) - \cos\left(\dfrac{3\pi}{2}\right)}{h}$?

(A) 1
(B) $\dfrac{\sqrt{2}}{2}$
(C) 0
(D) -1
(E) The limit does not exist.

2. At which of the five points on the graph in the figure at the right are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?



(A) A
(B) B
(C) C
(D) D
(E) E

3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

(A) $-\dfrac{3}{2}$
(B) -1
(C) $-\dfrac{5}{14}$
(D) $-\dfrac{3}{14}$
(E) 0

4. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

(A) $2\pi \int_0^1 r f(r)\, dr$

(B) $2\pi \int_0^1 r\,(1 + f(r))\, dr$

(C) $2\pi \int_0^2 r\,(1 + f(r))\, dr$

(D) $2\pi \int_1^2 r f(r)\, dr$

(E) $2\pi \int_1^2 r\,(1 + f(r))\, dr$

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

7. The solution to the differential equation $\frac{dy}{dx} = \frac{x^3}{y^2}$, where $y(2) = 3$, is

(A) $y = \sqrt[3]{\frac{3}{4}x^4}$

(B) $y = \sqrt[3]{\frac{3}{4}x^4} + \sqrt[3]{15}$

(C) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$

(D) $y = \sqrt[3]{\frac{3}{4}x^4 + 5}$

(E) $y = \sqrt[3]{\frac{3}{4}x^4 + 15}$

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval [0, 3]?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

9. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only
(B) $t > 4$ only
(C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$
(E) $3 < t < 4$ and $t > 5$

10. $\int (x - 1)\sqrt{x}\, dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$

(B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

(C) $\frac{1}{2}x^2 - x + C$

(D) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

(E) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

11. What is $\lim_{x \to \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

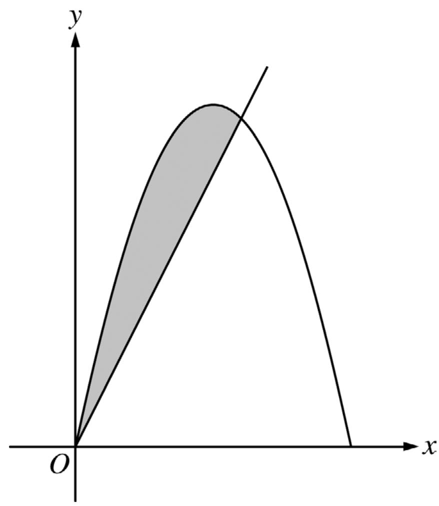
(A) -2

(B) $-\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) The limit does not exist.



12. The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$

(B) $\frac{9}{2}$

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

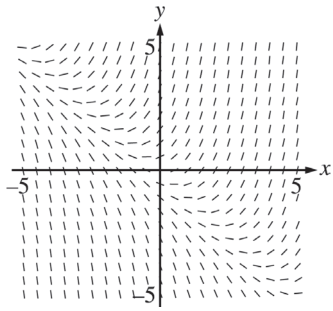
13. If f is a function that is continuous for all real numbers, then

$$\frac{d}{dx}\int_0^{x^2} f(t)\,dt =$$

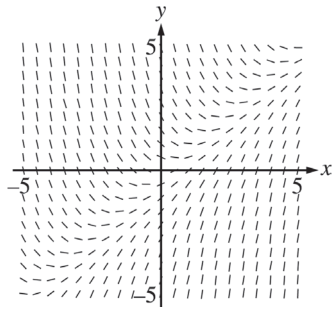
(A) $2x\,f(x^2)$
(B) $2x\,f(2x)$
(C) $f(2x)$
(D) $f(x^2)$
(E) $f'(x^2)$

14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

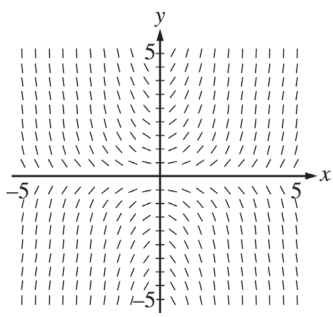
(A)



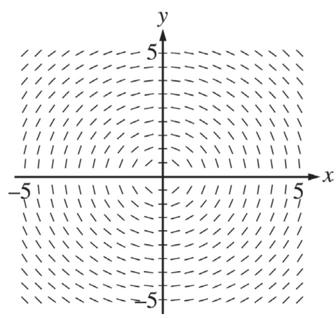
(B)



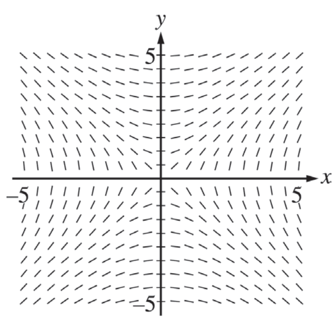
(C)



(D)



(E)



Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)} \sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

16. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(A) $\pi\int_0^1 \left(\left(3 - 2x^2\right)^2 - \left(3 - 2x\right)^2\right) dx$

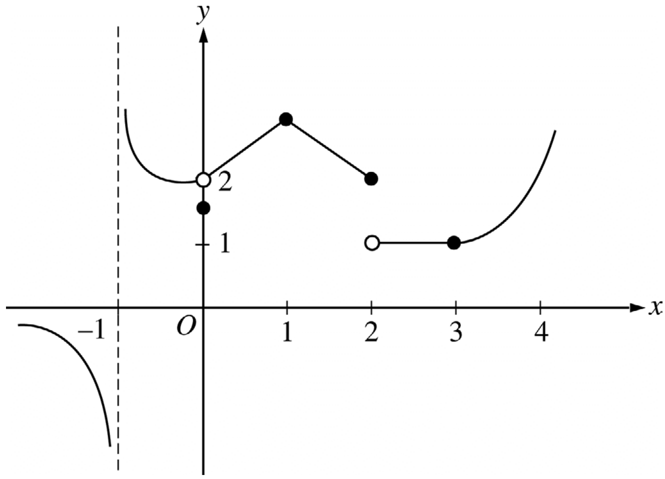
(B) $\pi\int_0^1 \left(\left(3 - 2x\right)^2 - \left(3 - 2x^2\right)^2\right) dx$

(C) $\pi\int_0^1 \left(4x^2 - 4x^4\right) dx$

(D) $\pi\int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2\right) dy$

(E) $\pi\int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2\right) dy$

17.



The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) –1
(B) 0
(C) 1
(D) 2
(E) 3

18.

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

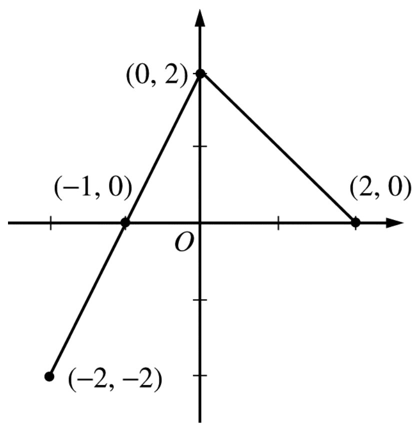
Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[1, 2]$. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x-axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

20.



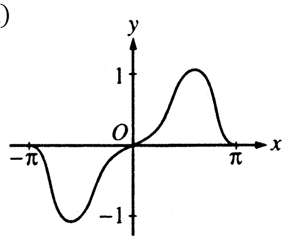
Graph of f

The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)\,dt$, then $g(-1) =$

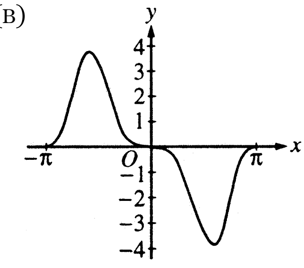
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

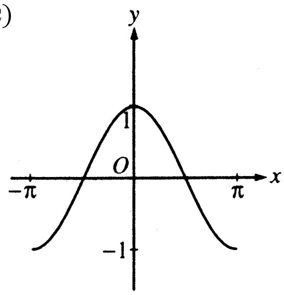
(A)



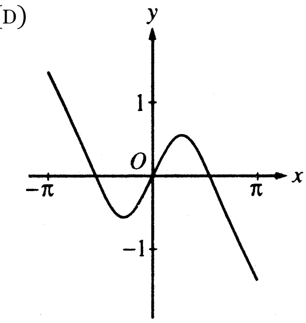
(B)



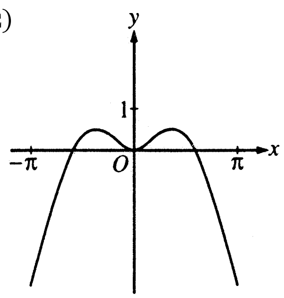
(C)



(D)



(E)



22. The base of a solid S is the semicircular region enclosed by the graph of $y = \sqrt{4 - x^2}$ and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{32\pi}{3}$

(B) $\frac{16\pi}{3}$

(C) $\frac{40}{3}$

(D) $\frac{32}{3}$

(E) $\frac{16}{3}$

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

(A) -1.819
(B) -0.843
(C) -0.819
(D) 0.157
(E) 1.157

Answers to Calculus AB Multiple-Choice Questions

Part A

1.	A	6.	B	11.	B
2.	B	7.	E	12.	B
3.	C	8.	C	13.	A
4.	D	9.	E	14.	E
5.	C	10.	D		

Part B

15.*	D	19.*	D	22.	D
16.	A	20.	B	23.*	D
17.	B	21.	E	24.*	E
18.	D				

* Indicates a graphing calculator-active question.

Calculus BC: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus BC are included in the following sections. Answers to the sample questions are given on page 43.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:
(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

   (A) $2x - 3y = 0$
   (B) $4x - 5y = 2$
   (C) $4x - y = 10$
   (D) $5x - 4y = 7$
   (E) $5x - y = 13$

2. If $3x^2 + 2xy + y^2 = 1$, then $\dfrac{dy}{dx} =$

(A) $-\dfrac{3x + y}{y^2}$

(B) $-\dfrac{3x + y}{x + y}$

(C) $\dfrac{1 - 3x - y}{x + y}$

(D) $-\dfrac{3x}{1 + y}$

(E) $-\dfrac{3x}{x + y}$

3.

x	$g'(x)$
-1.0	2
-0.5	4
0.0	3
0.5	1
1.0	0
1.5	-3
2.0	-6

The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) -6.5
(B) -1.5
(C) 1.5
(D) 2.5
(E) 3

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

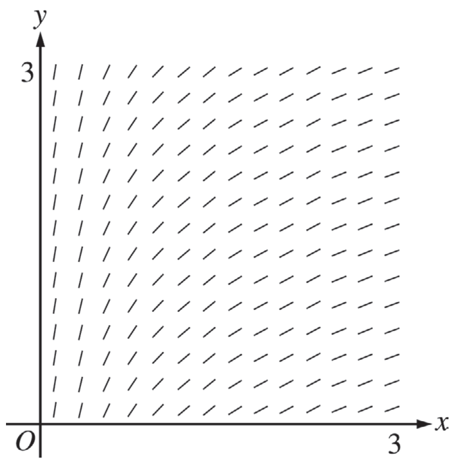
(A) All x except $x = 0$
(B) $|x| = 3$
(C) $-3 \le x \le 3$
(D) $|x| > 3$
(E) The series diverges for all x.

5. If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$

(A) $g(x^2)$
(B) $2xg(x)$
(C) $g'(x)$
(D) $2xg(x^2)$
(E) $x^2 g(x^2)$

6. If F' is a continuous function for all real x, then $\lim_{h \to 0} \frac{1}{h} \int_a^{a+h} F'(x)\,dx$ is

(A) 0
(B) $F(0)$
(C) $F(a)$
(D) $F'(0)$
(E) $F'(a)$



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

8. $\displaystyle\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\frac{3}{2}$
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) $\frac{3}{2}$
(E) divergent

9. Which of the following series converge to 2?

   I. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n+3}$

   II. $\displaystyle\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

   III. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) II and III only

10. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

   (A) 3
   (B) $3^{1/2}$
   (C) $18^{1/3}$
   (D) $36^{1/4}$
   (E) $36^{1/3}$

11. Which of the following integrals gives the length of the graph $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?

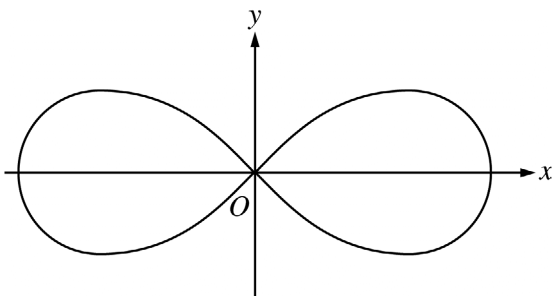
   (A) $\displaystyle\int_a^b \sqrt{x + \cos^2(\sqrt{x})}\, dx$

   (B) $\displaystyle\int_a^b \sqrt{1 + \cos^2(\sqrt{x})}\, dx$

   (C) $\displaystyle\int_a^b \sqrt{\sin^2(\sqrt{x}) + \frac{1}{4x}\cos^2(\sqrt{x})}\, dx$

   (D) $\displaystyle\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2(\sqrt{x})}\, dx$

   (E) $\displaystyle\int_a^b \sqrt{\frac{1 + \cos^2(\sqrt{x})}{4x}}\, dx$



12. What is the area of the region enclosed by the lemniscate $r^2 = 18 \cos(2\theta)$ shown in the figure above?

(A) $\frac{9}{2}$
(B) 9
(C) 18
(D) 24
(E) 36

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

14. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$
(B) $e^{\tan x} + 5$
(C) $5e^{\tan x}$
(D) $\tan x + 5$
(E) $\tan x + 5e^x$

Part B Sample Multiple-Choice Questions

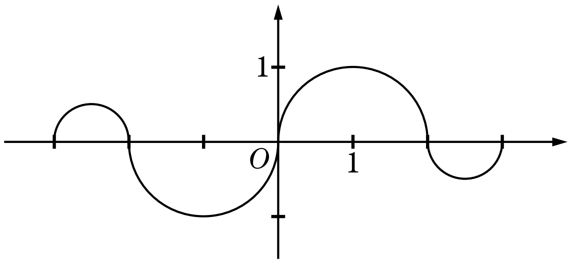
A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15.



Graph of f

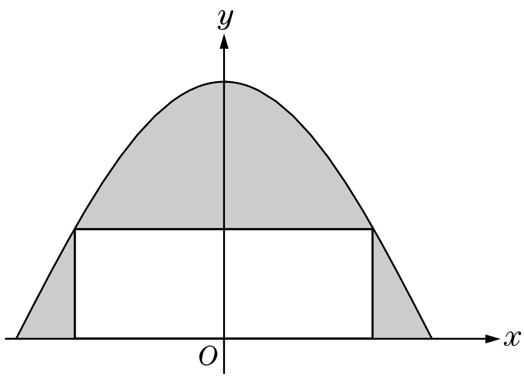
The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)dt$, where is $g(x)$ nonnegative?

(A) $[-3, 3]$
(B) $[-3, -2] \cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

16. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.
(B) $\lim_{x \to a} f(x)$ exists.
(C) $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists.
(D) $f'(a)$ is defined.
(E) $f''(a)$ is defined.

17.



A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

18. A solid has a rectangular base that lies in the first quadrant and is bounded by the x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid above the point (x, y) is $1 + 3x$. Which of the following is a Riemann sum approximation for the volume of the solid?

(A) $\displaystyle\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

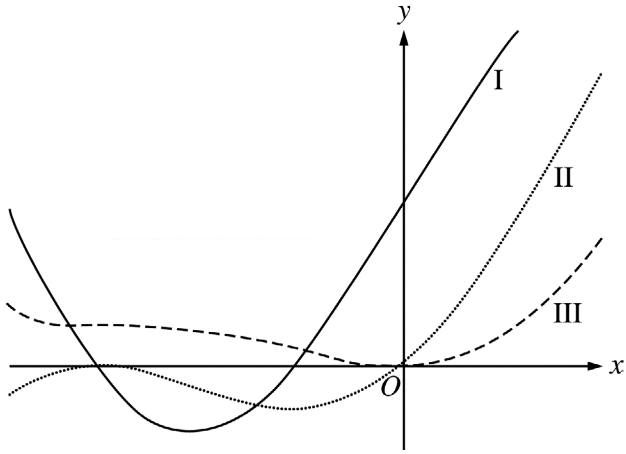
(B) $\displaystyle 2\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(C) $\displaystyle 2\sum_{i=1}^{n} \frac{i}{n}\left(1 + \frac{3i}{n}\right)$

(D) $\displaystyle\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right)$

(E) $\displaystyle\sum_{i=1}^{n} \frac{2i}{n}\left(1 + \frac{6i}{n}\right)$

19.



Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

20. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t + 1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) -3.268
(B) -1.585
(C) 1.732
(D) 6.585
(E) 11.585

22. Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2}\,dt$.

Which of the following statements about g must be true?

I. g is increasing on $(1, 2)$.
II. g is increasing on $(2, 3)$.
III. $g(3) > 0$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

23. A point (x, y) is moving along a curve $y = f(x)$. At the instant when the slope of the curve is $-\frac{1}{3}$, the x-coordinate of the point is increasing at the rate of 5 units per second. The rate of change, in units per second, of the y-coordinate of the point is

(A) $-\frac{5}{3}$
(B) $-\frac{1}{3}$
(C) $\frac{1}{3}$
(D) $\frac{3}{5}$
(E) $\frac{5}{3}$

24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.

For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
(B) 2.017
(C) 3.106
(D) 5.965
(E) 8.000

Answers to Calculus BC Multiple-Choice Questions

Part A

1. D	6. E	11. D
2. B	7. E	12. C
3. D	8. E	13. A
4. D	9. E	14. C
5. D	10. E	

Part B

15. A	19. E	22.* B
16. E	20.* C	23. A
17.* B	21.* B	24.* B
18. D		

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* Indicates a graphing calculator-active question.

Calculus AB and Calculus BC: Section II

Section II consists of six free-response problems. The problems do NOT appear in the Section II test booklet. Part A problems are printed in the green insert only; Part B problems are printed in a separate sealed blue insert. Each part of every problem has a designated workspace in the exam booklet. ALL WORK MUST BE SHOWN IN THE EXAM BOOKLET. (For students taking the exam at an alternate administration, the Part A problems are printed in the exam booklet only; the Part B problems appear in a separate sealed insert.)

The instructions below are from the 2005 exams. The free-response problems are from the 2004 exams and include information on scoring. Additional sample questions can be found at AP Central.

Instructions for Section II

PART A
(A graphing calculator is required for some problems or parts of problems.)
Part A: 45 minutes, 3 problems

During the timed portion for Part A, you may work only on the problems in Part A. The problems for Part A are printed in the green insert only. When you are told to begin, open your booklet, carefully tear out the green insert, and write your solution to each part of each problem in the space provided for that part in the pink exam booklet.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

PART B
(No calculator is allowed for these problems.)
Part B: 45 minutes, 3 problems

The problems for Part B are printed in the blue insert only. When you are told to begin, open the blue insert, and write your solution to each part of each problem in the space provided for that part in the pink exam booklet. During the timed portion for Part B, you may keep the green insert and continue to work on the problems in Part A without the use of any calculator.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B

For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM WITH A PENCIL OR PEN IN THE SPACE PROVIDED FOR THAT PART IN THE PINK EXAM BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

For more information on the instructions for the free-response sections, read the "Calculus FRQ Instruction Commentary" written by the AP Calculus Development Committee and the Chief Reader. It is available on the home pages for Calculus AB and Calculus BC at AP Central.

Beginning with the 2005 exams, sign charts by themselves will not be accepted as a sufficient response when a free-response question requires a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more detailed information on this policy change, read the article "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute Extrema" written by the AP Calculus Development Committee chair and the Chief Reader. It is available on the home pages for Calculus AB and Calculus BC at AP Central.

Calculus AB Sample Free-Response Questions

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

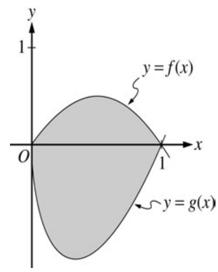
(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0,$ the traffic flow is decreasing at $t = 7.$	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

Question 2



Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) Area $= \int_0^1 (f(x) - g(x))\,dx$

$= \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right) dx = 1.133$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^1 \left((2 - g(x))^2 - (2 - f(x))^2\right) dx$

$= \pi \int_0^1 \left(\left(2 - 3(x-1)\sqrt{x}\right)^2 - (2 - 2x(1-x))^2\right) dx$

$= 16.179$

$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^1 (h(x) - g(x))^2\,dx$

$\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx = 15$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Question 3

A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

(a) $a(2) = v'(2) = -0.132$ or -0.133	1 : answer
(b) $v(2) = -0.436$ Speed is increasing since $a(2) < 0$ and $v(2) < 0$.	1 : answer with reason
(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$ $t = \ln(\tan(1)) = 0.443$ is the only critical value for y. $v(t) > 0$ for $0 < t < \ln(\tan(1))$ $v(t) < 0$ for $t > \ln(\tan(1))$ $y(t)$ has an absolute maximum at $t = 0.443$.	$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{identifies } t = 0.443 \text{ as a candidate} \\ 1 : \text{justifies absolute maximum} \end{cases}$
(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361 The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.	$4 : \begin{cases} 1 : \int_0^2 v(t)\,dt \\ 1 : \text{handles initial condition} \\ 1 : \text{value of } y(2) \\ 1 : \text{answer with reason} \end{cases}$

Question 4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

(a) $2x + 8yy' = 3y + 3xy'$ $(8y - 3x)y' = 3y - 2x$ $y' = \dfrac{3y - 2x}{8y - 3x}$	$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{solves for } y' \end{cases}$
(b) $\dfrac{3y - 2x}{8y - 3x} = 0;\ 3y - 2x = 0$ When $x = 3,\ 3y = 6$ $y = 2$ $3^2 + 4 \cdot 2^2 = 25$ and $7 + 3 \cdot 3 \cdot 2 = 25$ Therefore, $P = (3, 2)$ is on the curve and the slope is 0 at this point.	$3 : \begin{cases} 1 : \dfrac{dy}{dx} = 0 \\ 1 : \text{shows slope is 0 at } (3, 2) \\ 1 : \text{shows } (3, 2) \text{ lies on curve} \end{cases}$
(c) $\dfrac{d^2y}{dx^2} = \dfrac{(8y - 3x)(3y' - 2) - (3y - 2x)(8y' - 3)}{(8y - 3x)^2}$ At $P = (3, 2)$, $\dfrac{d^2y}{dx^2} = \dfrac{(16 - 9)(-2)}{(16 - 9)^2} = -\dfrac{2}{7}$. Since $y' = 0$ and $y'' < 0$ at P, the curve has a local maximum at P.	$4 : \begin{cases} 2 : \dfrac{d^2y}{dx^2} \\ 1 : \text{value of } \dfrac{d^2y}{dx^2} \text{ at } (3, 2) \\ 1 : \text{conclusion with justification} \end{cases}$

Question 5

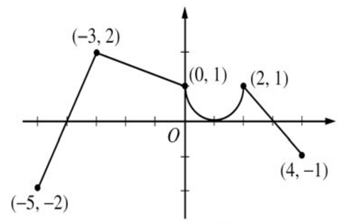
The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.



Graph of f

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$

$g'(0) = f(0) = 1$

$2 : \begin{cases} 1 : g(0) \\ 1 : g'(0) \end{cases}$

(b) g has a relative maximum at $x = 3$.
This is the only x-value where $g' = f$ changes from positive to negative.

$2 : \begin{cases} 1 : x = 3 \\ 1 : \text{justification} \end{cases}$

(c) The only x-value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints.

$g(-5) = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$

$g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13 - \pi}{2}$

So the absolute minimum value of g is -1.

$3 : \begin{cases} 1 : \text{identifies } x = -4 \text{ as a candidate} \\ 1 : g(-4) = -1 \\ 1 : \text{justification and answer} \end{cases}$

(d) $x = -3, 1, 2$

2 : correct values
$\langle -1 \rangle$ each missing or extra value

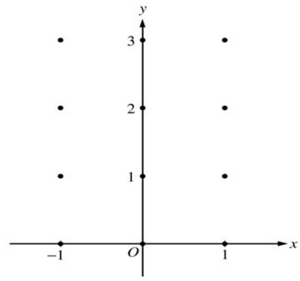
Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

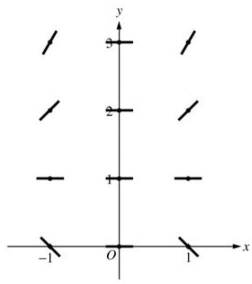
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(**Note: Use the axes provided in the pink test booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.



(a)



2 :
- 1 : zero slope at each point (x, y) where $x = 0$ or $y = 1$
- 1 :
  - positive slope at each point (x, y) where $x \neq 0$ and $y > 1$
  - negative slope at each point (x, y) where $x \neq 0$ and $y < 1$

(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

1 : description

(c) $\frac{1}{y-1}dy = x^2 dx$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$|y-1| = e^C e^{\frac{1}{3}x^3}$

$y - 1 = Ke^{\frac{1}{3}x^3}, K = \pm e^C$

$2 = Ke^0 = K$

$y = 1 + 2e^{\frac{1}{3}x^3}$

6 :
- 1 : separates variables
- 2 : antiderivatives
- 1 : constant of integration
- 1 : uses initial condition
- 1 : solves for y
  - 0/1 if y is not exponential

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

Calculus BC Sample Free-Response Questions

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

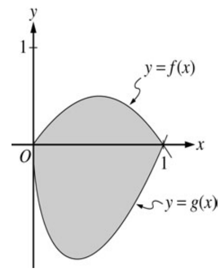
(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

Question 2

Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.



(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) Area $= \int_0^1 (f(x) - g(x))\, dx$

$= \int_0^1 \left(2x(1 - x) - 3(x - 1)\sqrt{x}\right) dx = 1.133$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^1 \left((2 - g(x))^2 - (2 - f(x))^2\right) dx$

$= \pi \int_0^1 \left(\left(2 - 3(x - 1)\sqrt{x}\right)^2 - (2 - 2x(1 - x))^2\right) dx$

$= 16.179$

$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^1 (h(x) - g(x))^2\, dx$

$\int_0^1 \left(kx(1 - x) - 3(x - 1)\sqrt{x}\right)^2 dx = 15$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Question 3

An object moving along a curve in the xy-plane has position $(x(t),\, y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2),\, y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t),\, y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a) $x(4) = x(2) + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$

$= 1 + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt = 7.132$ or 7.133

$3 : \begin{cases} 1 : \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$

$y - 8 = -2.983(x - 1)$

$2 : \begin{cases} 1 : \text{finds } \left.\frac{dy}{dx}\right|_{t=2} \\ 1 : \text{equation} \end{cases}$

(c) The speed of the object at time $t = 2$ is $\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382$ or 7.383.

1 : answer

(d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos\left(t^2\right)\right)$

$y''(4) = 24.813$ or 24.814

The acceleration vector at $t = 4$ is $\langle 2.303,\, 24.813 \rangle$ or $\langle 2.303,\, 24.814 \rangle$.

$3 : \begin{cases} 1 : x''(4) \\ 1 : \frac{dy}{dt} \\ 1 : \text{answer} \end{cases}$

Question 4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

(a) $2x + 8yy' = 3y + 3xy'$ $(8y - 3x)y' = 3y - 2x$ $y' = \dfrac{3y - 2x}{8y - 3x}$	$2: \begin{cases} 1: \text{implicit differentiation} \\ 1: \text{solves for } y' \end{cases}$
(b) $\dfrac{3y - 2x}{8y - 3x} = 0;\ 3y - 2x = 0$ When $x = 3,\ 3y = 6$ $y = 2$ $3^2 + 4 \cdot 2^2 = 25$ and $7 + 3 \cdot 3 \cdot 2 = 25$ Therefore, $P = (3, 2)$ is on the curve and the slope is 0 at this point.	$3: \begin{cases} 1: \dfrac{dy}{dx} = 0 \\ 1: \text{shows slope is 0 at } (3, 2) \\ 1: \text{shows } (3, 2) \text{ lies on curve} \end{cases}$
(c) $\dfrac{d^2y}{dx^2} = \dfrac{(8y - 3x)(3y' - 2) - (3y - 2x)(8y' - 3)}{(8y - 3x)^2}$ At $P = (3, 2)$, $\dfrac{d^2y}{dx^2} = \dfrac{(16 - 9)(-2)}{(16 - 9)^2} = -\dfrac{2}{7}$. Since $y' = 0$ and $y'' < 0$ at P, the curve has a local maximum at P.	$4: \begin{cases} 2: \dfrac{d^2y}{dx^2} \\ 1: \text{value of } \dfrac{d^2y}{dx^2} \text{ at } (3, 2) \\ 1: \text{conclusion with justification} \end{cases}$

Question 5

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

(a) For this logistic differential equation, the carrying capacity is 12. If $P(0) = 3$, $\lim_{t\to\infty} P(t) = 12$. If $P(0) = 20$, $\lim_{t\to\infty} P(t) = 12$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{answer} \end{cases}$
(b) The population is growing the fastest when P is half the carrying capacity. Therefore, P is growing the fastest when $P = 6$.	1 : answer
(c) $\frac{1}{Y}dY = \frac{1}{5}\left(1 - \frac{t}{12}\right)dt = \left(\frac{1}{5} - \frac{t}{60}\right)dt$ $\ln\lvert Y\rvert = \frac{t}{5} - \frac{t^2}{120} + C$ $Y(t) = Ke^{\frac{t}{5} - \frac{t^2}{120}}$ $K = 3$ $Y(t) = 3e^{\frac{t}{5} - \frac{t^2}{120}}$	$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } Y \\ \quad 0/1 \text{ if } Y \text{ is not exponential} \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables
(d) $\lim_{t\to\infty} Y(t) = 0$	1 : answer $0/1$ if Y is not exponential

Question 6

Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

(a) Find $P(x)$.

(b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.

(c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$.

(d) Let G be the function given by $G(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for G about $x = 0$.

(a) $f(0) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $f'(0) = 5\cos\left(\frac{\pi}{4}\right) = \frac{5\sqrt{2}}{2}$ $f''(0) = -25\sin\left(\frac{\pi}{4}\right) = -\frac{25\sqrt{2}}{2}$ $f'''(0) = -125\cos\left(\frac{\pi}{4}\right) = -\frac{125\sqrt{2}}{2}$ $P(x) = \frac{\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}x - \frac{25\sqrt{2}}{2(2!)}x^2 - \frac{125\sqrt{2}}{2(3!)}x^3$	4 : $P(x)$ $\langle -1 \rangle$ each error or missing term deduct only once for $\sin\left(\frac{\pi}{4}\right)$ evaluation error deduct only once for $\cos\left(\frac{\pi}{4}\right)$ evaluation error $\langle -1 \rangle$ max for all extra terms, $+\cdots$, misuse of equality
(b) $\frac{-5^{22}\sqrt{2}}{2(22!)}$	2 : $\begin{cases} 1 : \text{magnitude} \\ 1 : \text{sign} \end{cases}$
(c) $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| \le \max\limits_{0 \le c \le \frac{1}{10}} \left|f^{(4)}(c)\right| \left(\frac{1}{4!}\right)\left(\frac{1}{10}\right)^4$ $\le \frac{625}{4!}\left(\frac{1}{10}\right)^4 = \frac{1}{384} < \frac{1}{100}$	1 : error bound in an appropriate inequality
(d) The third-degree Taylor polynomial for G about $x = 0$ is $\int_0^x \left(\frac{\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}t - \frac{25\sqrt{2}}{4}t^2\right) dt$ $= \frac{\sqrt{2}}{2}x + \frac{5\sqrt{2}}{4}x^2 - \frac{25\sqrt{2}}{12}x^3$	2 : third-degree Taylor polynomial for G about $x = 0$ $\langle -1 \rangle$ each incorrect or missing term $\langle -1 \rangle$ max for all extra terms, $+\cdots$, misuse of equality