AP Calculus BC: Openers for Chapter 1 (Review)

1. Consider the function $f(x) = x + 2 + \frac{1}{10^{20}x}$.

   (a) Draw the graph of f. Is this a complete graph of f?
   (b) What appears to be the limit of $f(x)$ as x approaches zero?
   (c) Use the **Table** feature of your calculator to investigate what happens to $y = f(x)$ near $x = 0$.
   (d) Use the **Zoom** feature of your calculator to investigate what happens to $y = f(x)$ near $x = 0$. Set the **Zoom Factors** to ***x*Fact** of 100 and ***y*Fact** of 10. Turn the axes off. Start with a window of $x \in [-10, 10]$ and $y \in [-5, 5]$ and keep zooming in. Make sure that you use at $x = 0$, $y = 2$ as the center of your zooming.
   (e) What is the domain of f? Why?
   (f) What is the local behavior of the function near $x = 0$? What is the end behavior of this function?
   (g) Evaluate $f(1\times 10^{-14})$, $f(1\times 10^{-20})$, and $f(1\times 10^{-30})$.
   (h) What does this tell you about your calculator?

2. Let two functions f and g be defined by the tables below.

x	-5	3	5	10	15	20
$f(x)$	6	7	18	-5	-8	15

x	-5	6	5	10	15	18
$g(x)$	10	20	15	0	-5	-12

   (a) Make a table for $f \circ g$.

   (b) Make a table for $g \circ f$.

   (c) Make a table for $f \circ f$.

   (d) Make a table for $g \circ g$.

3. Consider $f(x)=\sqrt{4-x^2}$ and $g(x)=\lfloor x \rfloor$.

(a) What is a formula for $f\left(g\left(x\right)\right)$?

(b) What is the domain of $f\left(g\left(x\right)\right)$?

(c) What is the range of $f\left(g\left(x\right)\right)$?

4. Find f^{-1} for each function given below.

(i) $f(x)=3x+2$

(ii) $f(x)=4e^x$

(iii) $f(x)=\cos(x),\ x\in\left[0,\pi\right]$

(iv) $f(x)=x+\sin(x)$

Explain why the function *f*, given by $f(x)=x^2$ does not have an f^{-1}.
How could you restrict the domain of *f* so that f^{-1} would exist?

5. The functions $f(x)=x^3$ and $g(x) = \sqrt[3]{x}$ are inverse functions.

(a) Compute $\left(f\circ g\right)(x)$ and $\left(g\circ f\right)(x)$. What do you notice? Generalize.

(b) Draw the graphs of *f*, *g*, and the line $y = x$. What do you notice? Generalize.

(c) Prove analytically that *f* and *g* are inverse functions.

6. The functions $f(x)=\log(x)$ and $g(x)=10^x$ are inverse functions.

(a) Show that *f* and *g* are inverses by graphing them and the line $y = x$.

(b) Graph $(f \circ g)(x)$ with $x \in [-10,10]$, $y \in [-10,10]$. Why is this graph the same as the graph of $y = x$?

(c) Graph $(g \circ f)(x)$ with $x \in [-10,10]$, $y \in [-10,10]$. Why does this graph differ from the graph in (b)?

7. Let g be an odd function. Some values of g are shown in the following table. Fill in as many of the missing entries as possible.

x	-5	-2	1	3	4
$g(x)$	6	3	2	-1	-5
$g^{-1}(x)$					

8. Suppose that the line $y = 2$ is a horizontal asymptote of the function *f*. Find the horizontal asymptote of the functions given:

$g(x) = f(x)+3$

$g(x) = f(x+1)$

$g(x) = 3f(x+4)+1$

9. The polynomial function defined by $f(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ is a good approximation to the trigonometric function defined by $g(x) = \sin(x)$ provided x is fairly close to $x = 0$.

(a) Show that the statement above is true by using your graphing calculator.

(b) How much error is there between the value of g and the value of f for $x = 1.5$?

(c) For what values of x can the polynomial be used as an approximation for $\sin(x)$ and not have an error of more than .01?

10. Determine the range of each function.

(a) $f(x) = \frac{x}{x^2 - 4}$

(b) $f(x) = \frac{4x^2 - 12}{x^2 + 4}$

(c) $f(x) = \frac{\sin(x)}{2x}, \quad x \in [-\pi, \pi]$

(d) $f(x) = e^{\sin(x)}$ (Try this analytically.)

11. Arrange each of the following in terms of the size of $f(x)$ as x gets to be a very large positive number.

(a) $f(x) = 2x - 5$

(b) $g(x) = 4\log(x)$

(c) $h(x) = \frac{1}{10}2^x$

(d) $k(x) = x^5 - 3x$

12. For each of the functions defined in problem number 3, determine what happens as x gets to be a very large number for each of the following.

(a) $\frac{f(x)}{k(x)}$

(b) $\frac{f(x)}{g(x)}$

(c) $\frac{f(x)}{h(x)}$

(d) $\frac{k(x)}{h(x)}$

(e) $\frac{f(x) \cdot g(x)}{k(x)}$

(f) $\frac{h(x)}{g(x)}$

13. Suppose that $y = f(x)$ is an exponential function, i. e., $f(x) = ab^x$, where a and b are constants and $b > 0$.

(a) Explain why $\dfrac{f(5)}{f(3)} = \dfrac{f(10)}{f(8)}$.

(b) Find a number a such that $\dfrac{f(4)}{f(1)} = \dfrac{f(a)}{f(-9)}$.

(c) Explain why $\left(\dfrac{f(2)}{f(-1)}\right)^3 = \dfrac{f(10)}{f(1)}$.

(d) Show that the expression $\dfrac{f(x+h)}{f(x)}$ does not depend on x.

(e) The value of the expression in part (d) does not depend on the value of x. Show that the value of the expression $\dfrac{\ln\left(\dfrac{f(x+h)}{f(x)}\right)}{h}$ does not depend on h. (Assume $h \neq 0$.

14. Explain the effect of changing the base, assuming the base is always positive, on the graph of a logarithmic function. Do the same for the base of an exponential function.

15. Given the function f, defined by $f(x) = a\sin(bx + c) + d$, explain the effect on the graph of f in changing each of the parameters a, b, c, and d.