

AP Calculus BC

Lesson 10.1 Differential Equations

10.1(1) For each differential equation given below, solve for the general solution:

a)  $\frac{dy}{dx} = y$

b)  $\frac{dy}{dx} = \frac{1}{x}$

c)  $\frac{dy}{dx} = \cos(2x)$

10.1(2) For each of the differential equations given in 12.1(1) find the particular solution that satisfies the initial condition  $y(1) = 4$ .

10.1(3)

a) Show that  $y = A \sin(x) + B \cos(x)$  is a solution of the second order differential equation  $\frac{d^2y}{dx^2} = -y$  for any constants  $A$  and  $B$ .

b) Make a graph of the solutions to  $\frac{d^2y}{dx^2} = -y$  for the following values of  $A$  and  $B$ .

i)  $A = 1, B = 0$

ii)  $A = 0, B = 1$

iii)  $A = 1, B = 1$

iv)  $A = -1, B = -1$

v)  $A = -1, B = 0$

vi)  $A = 0, B = -1$

10.1(4)

a) Show that  $y = Ke^{-0.1t} + 70$  is a solution of the differential equation  $\frac{dy}{dx} = -0.1(y - 70)$ .

b) Graph a family of solutions to this differential equation where  $K$  varies from  $-5$  to  $5$  in steps of one unit.

10.1(5) The rate at which you learn new material is directly proportional to the product of the amount you have learned and the difference between the amount you have learned and 90% of the total amount there is to learn.

a) Write a differential equation for this statement.

b) Show that  $L = \frac{.90C}{de^{-.90Ckt} + 1}$  satisfies the differential equation given in part a), where  $L$  is the amount learned at time  $t$  minutes and  $C$  is the total amount you have to learn and  $d$  and  $k$  are some constants.

c) Suppose you are studying for a 100 question test. If you study for an exceedingly long time, what is the most number of questions you can expect to get correct?

d) If at time  $t = 0$  you could answer 3 questions correctly, and at time  $t = 60$  minutes you can answer 50 questions correctly, for how many minutes must you study in order to get 75 of the test answered correctly?