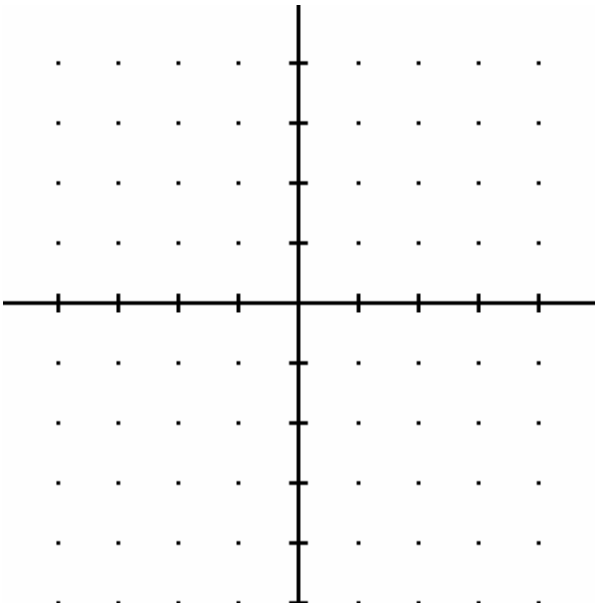
AP Calculus BC
Lesson 10.2 Slope Fields

10.2(1) Consider the differential equation $\frac{dy}{dx} = -\frac{x}{y}$. Draw a small line segment at each lattice point of the form (*a*,*b*), where $|a| \le 4$ and $|b| \le 4$, showing the slope of the curve there. Sketch a possible solution for this differential equation that passes through the point (1,2).



10.2(2) To recreate problem 1 on your calculator, do the following:

Press MODE and change Graphing mode to DIFF EQUATIONS.

On the Y= menu, enter $y1' = \frac{-t}{y1}$.

Go to Window (green F2) and set xmin = -6, xmax = 6, ymin = -3, ymax = 3.
Make sure that ncurves = 0.

Press Graph (green F3) to see the slopefield.

To see the curve with a specific initial condition, go back to the Y= menu and enter the initial condition t0 = 1 and yi1 = 2. Then press Graph to see the curve. ☺

Notes about entering differential equations:
Do not use function notation! Use only y1 (not y1(t)) Likewise, use multiplication symbols in front of expressions in parentheses to avoid confusion with function notation.

10.2(3)Use your calculator to construct slope fields for the following differential equations. Describe the solution for each differential equation that passes through the given point.

(a) $\frac{dy}{dt} = \frac{-y}{t}$ point (4,1)

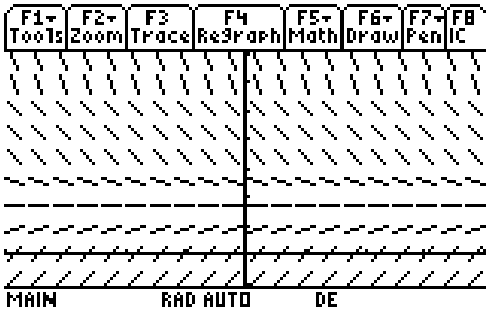
(b) $\frac{dy}{dt} = .01(70 - y)$ point (0,100) *(change the window!)*

(c) $\frac{dy}{dt} = ty$ point (1,1)

(d) $\frac{dy}{dt} = .01y(50 - y)$ point (2,2) *(change the window!)*

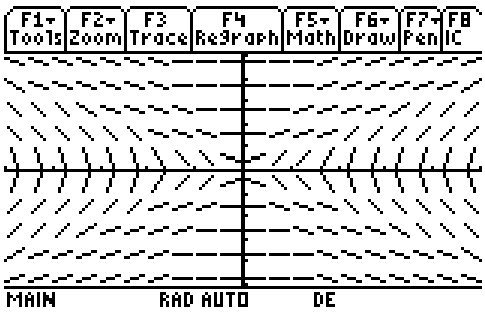
10.2(4) The slope fields for $\frac{dy}{dt} = 2 - y$ and $\frac{dy}{dt} = \frac{t}{y}$ are shown below. For each slope field:

(a) Sketch solution curves with initial conditions
(i) $y = 1$ when $t = 0$
(ii) $y = 0$ when $t = 1$
(iii) $y = 3$ when $t = 0$



$-4 \le x \le 4$, $-1 \le y \le 7$

Slope field for $\frac{dy}{dx} = 2 - y$



$-5 \le x \le 5$, $-5 \le y \le 5$

Slope field for $\frac{dy}{dt} = \frac{t}{y}$

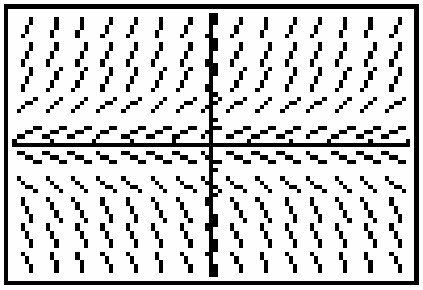
(b) For each solution curve, can you say anything about the long-run behavior of y? For example, does $\lim_{t \to \infty} y$ exist? If so, what is its value?

10.(5) Match the slope fields below with their differential equations:

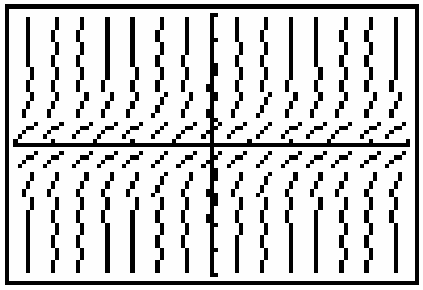
a. $y' = 1 + y^2$ b. $y' = x$ c. $y' = \sin(x)$

d. $y' = y$ e. $y' = x - y$ f. $y' = 4 - y$

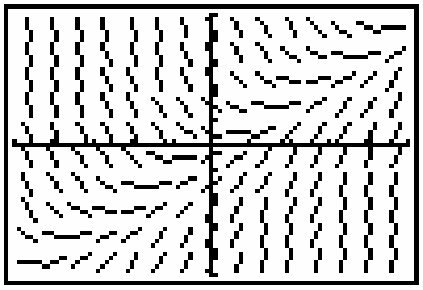
(i)



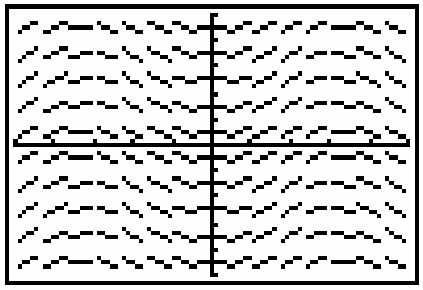
(ii)



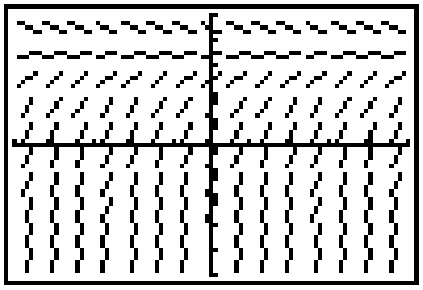
(iii)



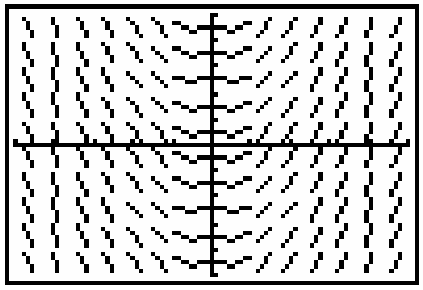
(iv)



(v)



(vi)



10.2(6) Consider the differential equation $\frac{dy}{dx} = 1 + y^2$.

(a) Construct the family of solution curves using your calculator.
(b) Sketch the curve that passes through (0,0). What function is this?

10.2(7) Consider the differential equation $\frac{dy}{dx} = e^{-x^2}$.

(a) Construct the slopefield using your calculator.
(b) Use your calculator to sketch the solution curve that goes through (0,0).

10.2(8)

Consider $x^2 + xy = 4$. Slope fields give a way to see the shape of this relation near say (1,3).

(a) Find $\frac{dy}{dx}$.

(b) Sketch the slope field for this relation.
Make sure you have (1,3) in your viewing window.