

AP Calculus BC

Lesson 10.2 continued Euler's Method

- To see how Euler's method works, consider the following IVP.

$$\frac{dy}{dx} = 3x^2 \text{ and } y(-2) = -6$$

Euler's method picks a step size and pretends that over each interval of that step size the curve is essentially linear, with slope given by differential equation at the left endpoint. In our example, suppose we pick a step size of .5, use 10 points, not including the initial point, and start at our initial condition point of (-2,-6). Complete the following table:

Initial  $x = -2$

Initial  $y = -6$

Step size = 0.5

Number of points = 10

$x$	-2	-1.5	-1	-.5	0	.5	1	1.5	2	2.5	3
$y$	-6	0	3.375								
slope	12	6.75	3								

- Now solve the differential equation analytically and look at the actual values of  $y$ . How do they compare with those in the table, and why?
- Use your calculator to draw the slope field and the specific solution through the point (-2, -6). (Don't forget to spell " $x$ " as " $t$ " and to be in Differential Equation Mode.)

Change the following settings:

tstep = .5, xmin = -3, xmax = 4, ymin = -10, ymax = 30

Press F1 and choose **9: format** and change **solution method** from RK to **Euler**.

Press graph to see the solution.

Press F3 to trace the coordinates from the graph. The coordinates are also available in the table (F5) – make sure to set up (F4) the table appropriately.

To graph the actual solution, press F6 and choose 2: DrawFunc. The command **DrawFunc** appears on the homescreen. Type the rule (*no  $y =$  or  $f(x) =$* ) and press enter to see the graph. (for example, **DrawFunc sin(x)** will draw the graph of  $y = \sin(x)$ .)

4. Suppose that we know the following facts about the world population.
- The world population in 1990 was approximately 5.333 billion.
  - The world population changes at a rate proportional to the current population.
  - On the average, the constant of proportionality is approximately 0.017 (based on historical population data).

How can we use this information to predict the world population in the years following 1990? What will be the world population in 1998?

- a. Use a linear approximation to estimate the population in 1998.
- b. Use a linear approximation to estimate the population in 1994. Use that estimate with a linear approximation to estimate the population in 1998.
- c. Use your calculator with the following step sizes to estimate the population in 1998:  
(i) 8                      (ii) 4                      (iii) 1
- d. Solve the differential equation.
- e. Estimate the population in 2006 using step sizes of 1.
- f. Estimate the population in 2190 using step sizes of 10.
5. Consider the function that satisfies the differential equation  $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$  and goes through the point (1,2). Do the following:
- (a) Use Euler's method to approximate the value of  $y$  when  $x = 20$ .
- (b) Find the particular solution of this differential equation analytically.
- (c) What is the actual value of  $y$  when  $x = 20$ ?
- (d) Discuss the error in the estimation you made in part (a). Was your estimate too large or too small? Explain why.