AP Calculus BC
Lesson 10.3 Separable Differential Equations

10.3(1) Find the complete solution for each of the following differential equations.

(a) $\frac{dy}{dx} = 3xy^2$ (b) $\frac{dy}{dx} = 5\sqrt{y}$ (c) $\frac{dy}{dx} = (1+y^2)e^x$

(d) $\frac{dy}{dx} = \frac{\sec^2(x)}{\tan^2(y)}$ (e) $\frac{du}{dv} = \frac{3v\sqrt{1+u^2}}{u}$ (f) $\frac{du}{dv} = \frac{\cos(2v)}{\sin(3u)}$

10.3(2) Solve each of the equations in 10.3(1) with your calculator as follows:

On the Calculus Menu, choose C: **deSolve(**

The command takes three arguments: a differential equation (*use the ' key above the = sign*), the independent variable, and then the dependent variable.

Example: **deSolve(y'=3x*y^2, x, y)** solves part a).

Note that the constant of integration comes out as an @1 or @n for some other n.

If you wish to type the @ sign, use ♦ STO→

10.3(3)
Find the particular solution to each differential equation given below.

(a) $\frac{dy}{dx} = \frac{\cos(3x)}{\sin(2y)}$ and $y = \frac{\pi}{3}$, when $x = \frac{\pi}{2}$

(b) $\frac{d^2u}{dv^2} = 4(1+3v)^2$ and u = -1 and $\frac{du}{dv} = -2$, when v = -1

(c) $\frac{dy}{dx} = \frac{-x}{y}$ and $y(0) = 6$

(d) $\frac{dy}{dx} = \frac{1}{y\ln(y)}$ and $y(0) = 1$

Note: you can use your calculator to find the specific solution by adding the phrase "and $y(x_0) = y_0$" as follows:

deSolve(y'=cos(3x)/sin(2y) **and y(π/2)= π/3**, x, y) will give a solution to part a).

Notice that in several cases the format is not what you expect. ☺

You can also use the regular ***solve(*** *command to solve for the @1 constant.*

10.3(4) When a murder is committed, the body, originally at 37° C, cools according to Newton's Law of cooling, which has the differential equation $\frac{dH}{dt} = k(H - A)$, where H is the temperature of the body at time t and A is the ambient temperature, i.e., the temperature of the surroundings. Suppose that after 2 hours, the temperature has cooled to 35° C and that the temperature of the surroundings is 20° C.

(a) Find the temperature H of the body as a function of t, the time in hours since the murder was committed.

(b) Sketch a graph of temperature against time.

(c) What happens to the temperature in the long run? Show this on the graph and analytically.

(d) If the body is found at 4 PM at a temperature of 30° C, when was the murder committed?

10.3(5) (Dedicated to Mr. Dodge)

At 1:00 PM on a winter afternoon, there was a power failure at Mr. Dodge's home in northern Wisconsin and, of course, his furnace does not work without electricity. When the power went out it was 68° F in his home. At 10:00 PM, it was 51° F in the house and the temperature outside was a chilly -10° F.

(a) Assuming that the temperature T in my home obeys the Newton's law of cooling, write the differential equation satisfied by T.

(b) Solve the differential equation and use it to estimate the temperature in Mr. Dodge's home at 7:00 AM the next morning. Should he worry about his pipes freezing?

(c) What assumptions did you have to make in part (a) about the temperature outside? Given that this is probably an incorrect assumption, would you revise your estimate up or down? Why ?

10.3(6)

Find an equation of the curve that goes through the point $\left(\frac{\pi}{6},0\right)$ and for which the slope at any point (x,y) on it is given by $\frac{(2y-4)}{\tan(x)}$.