AP Calculus BC
Lesson 10.5 The Logistic Curve

10.5(1) Solve the differential equation by hand:

(a) $\frac{dy}{dx} = 4y(25 - y)$

(b) $\frac{dy}{dx} = 0.0125y(350 - y)$ and $y(0) = 30$

10.5(2) A certain national park is known to be able to support at most 100 grizzly bears. Ten bears are known to be in the park at the present. Assume that the rate of population growth is directly proportional to both the current population of bears and difference between 100 and the current population of bears, with proportionality constant 0.001.

(a) Determine the population of bears $y(t)$ as an explicit function of time t.

(b) Draw a graph of $y(t)$ from the present, for the next 100 years.

(c) When does the population of bears exceed 90% of the park's carrying capacity?

10.5(3) When there is no fishing on the Wisconsin body of water the Turtle Flambeau Flowage, suppose the population of fish is governed by the differential equation

$$\frac{dp}{dt} = 2p - 0.01p^2$$

where p is the number of fish in the body of water at time t in years. Suppose that in addition, that the fish are really removed by fishermen at the continuous rate of 75 fish/year.

(a) What is the differential equation that the fish population satisfies?

(b) Use your calculator to sketch solutions to the differential equation satisfying the initial conditions
(i) $P(0) = 40$ (ii) $P(0) = 50$ (iii) $P(0) = 60$
(iv) $P(0) = 150$ (v) $P(0) = 170$

(c) There are two equilibrium populations, what are they?

10.5(4) A rumor spreads through a community according to differential equation $\frac{dy}{dt} = 2y(1-y)$, where y is the proportion of the population that have heard the rumor at time t.

(a) What proportion has heard the rumor when it is spreading the fastest? Justify your answer.

(b) If at time t = 0 ten percent of the people have heard the rumor, find y as a function of t.

(c) At what time is the rumor spreading the fastest?

10.5(5) The following table will be used to collect data for a model to describe the spread of a disease. Let's suppose that 25 people are in a town and a visitor comes to the town bringing a serious disease. That person is the only one with the disease initially. On any given day an infected person can only infect one other person. Let's model the spread of the disease. We will use a random number generator to select the persons to be infected.

Day	**Number infected**
1	
2	
3	
4	
5	
6	
7	
8	
9	

(a) After collecting the data, make a scatterplot of the information. This curve is a *logistic* curve and has the form $\frac{dy}{dt} = ky(A - y),$ where y is the number infected, and A is the total population and k is some constant of proportionality. Find the solution logistic curve for our disease model

(b) Graph your solution to show that it fits the data.

(c) Describe in words why this differential equation makes sense to use with this model.