AP Calculus BC
Vector Functions: **Additional problems for Assignments #90-93**
DO ON SEPARATE PAPER! ☺

1. Find the domain of the function $\mathbf{r}(t) = \left\langle \sqrt{t-1}, \sqrt{5-t} \right\rangle$

2. Find the limit: $\lim_{t \to 0+} \mathbf{r}(t)$ if $\mathbf{r}(t) = \left\langle \cos t, t \ln t \right\rangle$

3. Find the limit: $\lim_{t \to 1} \mathbf{r}(t)$ if $\mathbf{r}(t) = \left\langle \frac{t-1}{t^2-1}, \frac{\tan t}{t} \right\rangle$

4. Sketch the curve given by $\mathbf{r}(t) = \left\langle t^4 + 1, t \right\rangle$. Indicate the direction of motion.

5. Sketch the curve given by $\mathbf{r}(t) = \left\langle t^3, t^2 \right\rangle$. Indicate the direction of motion.

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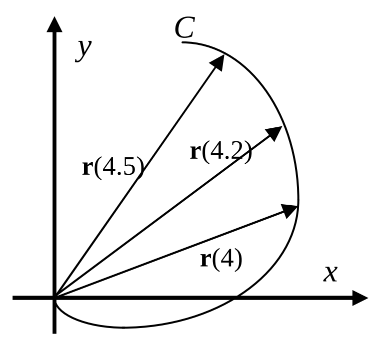
6. Find $\lim_{t \to 2} \mathbf{R}(t)$ if $\mathbf{R}(t) = (3t-2)\mathbf{i} + t^2\mathbf{j}$

7. The figure at right shows a curve C given by a vector function $\mathbf{r}(t)$.

   a) Draw the vectors $\mathbf{r}(4.5) - \mathbf{r}(4)$ and $\mathbf{r}(4.2) - \mathbf{r}(4)$

   b) Draw the vectors $\frac{\mathbf{r}(4.5) - \mathbf{r}(4)}{0.5}$ and $\frac{\mathbf{r}(4.2) - \mathbf{r}(4)}{0.2}$

   c) Write an expression for $\mathbf{r}'(4)$,
   and graph the tangent (velocity) vector.



8. a) Sketch the plane curve given by the vector equation $\mathbf{r}(t) = (1+t)\mathbf{i} + t^2\mathbf{j}$
   b) Find the velocity vector $\mathbf{r}'(t)$
   c) Sketch the position and velocity vectors at $t = 1$

9. a) Sketch the plane curve given by the vector equation $\mathbf{r}(t) = e^t\mathbf{i} + e^{3t}\mathbf{j}$
   b) Find the velocity vector $\mathbf{r}'(t)$
   c) Sketch the position and velocity vectors at $t = 0$

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10. Given the Vector-Valued Function $\mathbf{r}(t) = \langle 2t+4,\ t^2 \rangle$
    a) Find the domain of the function
    b) Find the value of the function when t = 2
    c) Find $|\mathbf{r}(t)|$ when t = 2
    d) Find the velocity vector $\mathbf{r}'(t)$
    e) Find the speed of the tip of the vector when t = 2
    f) Find the acceleration vector $\mathbf{r}''(t)$

11. Given $\mathbf{r} = (4\sin t)\mathbf{i} + (5 - 4\cos t)\mathbf{j}$
    a) Sketch the graph of this function
    b) Find the velocity vector
    c) Find the acceleration vector
    d) Add sketches of the velocity and acceleration vectors at $t = \pi$ to the graph of the function from part a).
    e) Find the slope of the function when $t = \frac{4\pi}{3}$

12. Given $\mathbf{r} = \ln(t+1)\mathbf{i} + (\sin^{-1} t)\mathbf{j}$
    a) Give the domain of the function
    b) Find the velocity and acceleration of this function
    c) Find the speed of the tip of the vector when t = 0.5
    d) Find $|\mathbf{r}(t)|$
    e) Find $\int \mathbf{r}(t)dt$
    f) Find $\int_0^1 \mathbf{r}(t)dt$

13. A particle moves around the ellipse $\frac{y^2}{9} + \frac{z^2}{4} = 1$ in the yz-plane such that it position at time *t* is given by $\mathbf{r}(t) = (3\cos t)\mathbf{j} + (2\sin t)\mathbf{k}$. Find the maximum and minimum values of $|\mathbf{v}|$ and $|\mathbf{a}|$.

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14. Solve the initial value problem for **r** as a vector function of *t*:

$$\frac{d\mathbf{r}}{dt} = \left(\tfrac{3}{2}\sqrt{t+1}\right)\mathbf{i} + \left(e^{-t}\right)\mathbf{j} + \left(\frac{1}{t+1}\right)\mathbf{k} \quad \text{and} \quad \mathbf{r}(0) = \mathbf{k}$$

15. Find $\frac{d}{dt}\left|\mathbf{R}(t)\right|$ if $\mathbf{R}(t)=\left(e^{t}+1\right)\mathbf{i}+\left(e^{t}-1\right)\mathbf{j}$

16. Find $\mathbf{R}'(t)\cdot\mathbf{R}''(t)$ if $\mathbf{R}(t)=\ln(t-1)\mathbf{i}+\left(-3t^{-1}\right)\mathbf{j}$

17. Show that a vector with constant length is always perpendicular to its velocity vector.

18. Find the angle between the functions $\mathbf{R}(t)=3e^{2t}\mathbf{i}-4e^{2t}\mathbf{j}$ and $\mathbf{R}(t)=6e^{3t}\mathbf{j}$.

19. Suppose a projectile is fired into the air at an initial velocity of 900 m/sec and at an angle of 30° (yes that's degrees!) from the ground.

    a) Write a vector equation that models the path of the projectile, using time t in seconds after launch as the parameter.

    b) Find the maximum height of the projectile.

    c) Find the range of the projectile.