AP Calculus BC

Assignments 99 and 100 - Review on Vector, Polar, and Parametric Functions

DO ON SEPARATE PAPER! ☺

1. Consider the parametric curve given by $x = 2t^3 - 1,\ y = 2t + 1$.

   (a) Find $\frac{dy}{dx}$ at $x = 7$.

   (b) Find $\frac{d^2y}{dx^2}$.

   (c) Give a Cartesian equation for the curve.

2. Find the arclength of the curve given by $x = t^2,\ y = t^3$, for $0 \le t \le 1$.

3. Write an equation for the line tangent to the polar curve $r = 1 + \sin(\theta)$ at the point where $\theta = \pi/3$.

4. Find the length of the polar curve $r = \frac{e^\theta}{\sqrt{2}}$ from $0 \le \theta \le \pi$.

5. Find the area inside the convex limaçon $r = 4 + 2\cos(\theta)$.

6. Find the area shared by the cardioids $r = 2 + 2\cos(\theta)$ and $r = 2 - 2\cos(\theta)$.

7. Find the area of the region bounded by $r = \sec(\theta)$, $\theta = 0$, and $\theta = \tan^{-1}(m)$.

8. A baseball is hit when it is 2.5 feet above the ground and leaves the bat with initial velocity of 145 ft/sec at a launch angle of 23°. A wind of 10 mph is blowing in the horizontal direction against the ball. A 15-foot high fence is 300 feet from home plate in the direction of the flight of the ball.

   (a) Write a vector equation which simulates the flight of the baseball.

   (b) How high does the baseball go, and when does it reach its maximum height?

   (c) Does the batter hit a home run? Explain.

9. The position of a particle at any time $t \ge 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

   (a) Find the magnitude of the velocity vector at $t = 5$.

   (b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

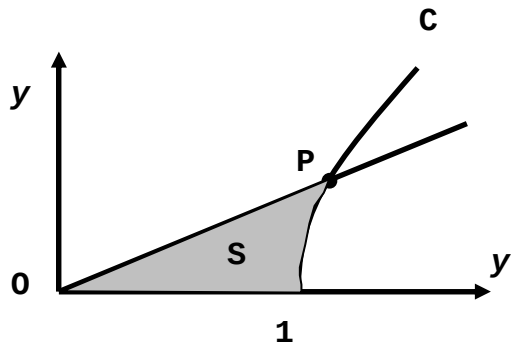
   (c) Find $\frac{dy}{dx}$ as a function of x.

10. [1987BC5] The position of a particle moving in the xy-plane at any time t, $0 \le t \le 2\pi$, is given by **R**(t) = sin(t)**i** + cos(2t)**j.**

    (a) Find the velocity vector for the particle for any time t, $0 \le t \le 2\pi$.
    (b) For what values of t is the particle at rest?
    (c) Write an equation for the path of the particle in terms of *x* and *y* that does not involve trigonometric functions.
    (d) Sketch the path of the particle in the xy-plane.

11. [1973BC4] A kite flies according to the parametric equations $x = \frac{t}{8}, y = -\frac{3}{64}t(t-128),$ where t is measured in seconds and $0 < t \le 90$.

    (a) How high is the kite above the ground at time t = 32 seconds?
    (b) At what rate is the kite rising at t = 32 seconds?
    (c) At what rate is the string being reeled out at t = 32 seconds?
    (d) At what time does the kite start to lose altitude?



12. (2003BC3) The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1+y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

    (a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for the curve C at point P.

    (b) Set up and evaluate an integral expression with respect to y that gives the area of S.

    (c) Curve C is part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

    (d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S.