AP Calculus BC
Review for chapter 11

1. (1974BC5) Given the parametric equations $x = 2\left(\theta - \sin\left(\theta\right)\right)$ and $y = 2\left(1 - \cos\left(\theta\right)\right)$.

   (a) Find $\frac{dy}{dx}$ in terms of θ.

   (b) Find an equation of the line tangent to the graph at $\theta = \pi$.

   (c) Find an equation of the line tangent to the graph at $\theta = 2\pi$.

   (d) Find the length of the curve defined by these equations over the interval $0 \le \theta \le 2\pi$.

2. (1975BC3) A particle moves on the circle $x^2 + y^2 = 1$ so that at time $t \ge 0$ is given by the vector $\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)$.

   (a) Find the velocity vector.

   (b) Is the particle ever at rest? Justify your answer.

   (c) Give the coordinates of the point that the particle approaches as *t* increases without bound.

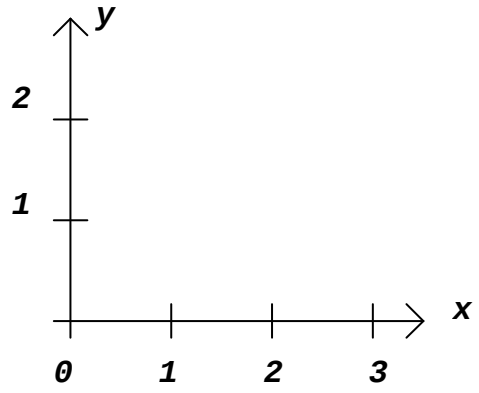
3. A particle moves in the *xy*-plane so that at any time $t \geq 0$ its position (x,y) is given by $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$.

(a) Find the velocity vector for any $t \geq 0$.

(b) Find $\lim_{t \to \infty} \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}}$.

(c) The particle moves on a hyperbola. Find an equation for this hyperbola in terms of x and y.

(d) On the axes provided, sketch the path of the particle showing the velocity vector for $t = 0$.

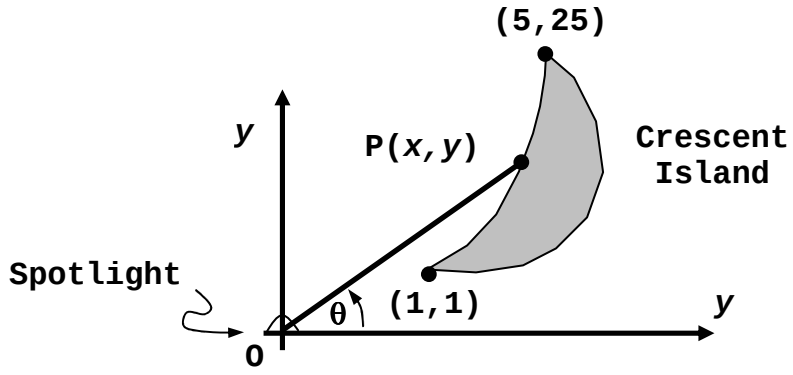


4. (1984BC2) The path of a particle is given for time $t > 0$ by the parametric equations $x = t + \frac{2}{t}$ and $y = 3t^2$.

(a) Find the coordinates of each point on the path where the velocity of the particle in the x direction is zero.

(b) Find $\frac{dy}{dx}$ when $t = 1$.

(c) Find $\frac{d^2y}{dx^2}$ when $y = 12$.



Note: Figure not drawn to scale.

5. (1996BC6) The figure above shows a spotlight shining on a point P(x,y) on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point (1,1) to the point (5,25). Let θ be the angle between the beam of light and the positive x-axis.

   (a) For what values of θ between 0 and 2π does the spotlight shine on the shoreline?

   (b) Find the x- and y-coordinates of point P in terms of $\tan\theta$.

   (c) If the spotlight is rotating at a rate of one revolution per minute, how fast is the point P traveling along the shoreline at the instant it is at the point (3,9)?