AP Calculus BC
Lesson 11.1 Parametric Equations and Vector Functions

1. For each set of parametric equations, identify the graph and give a Cartesian equation for the graph. Indicate the portion of the graph traced and the direction of motion.

   a) $x = \cos(2t)$, $y = \sin(2t)$, $0 \le t \le \pi$

   b) $x = 4\sin t$, $y = 5\cos t$, $0 \le t \le 2\pi$

   c) $x = \csc t$, $y = \cot t$, $0 < t < \pi$

2. Consider the vector function $\mathbf{R}(t) = (3t)\,\mathbf{i} + (1 - 9t^2)\,\mathbf{j} = \langle 3t, 1 - 9t^2 \rangle$

   (a) Find the domain of **R.**

   (b) Sketch the curve determined by **R.**

   (c) Find parametric equations for *x* and *y*.

   (d) Find a Cartesian equation for this curve.

3. Consider the parametric equations $\begin{cases} x(t) = \dfrac{t^2 - 1}{(t^2 + 1)} \\ y(t) = \dfrac{2t}{(t^2 + 1)} \end{cases}$

a) For what values of t is this parametric equation defined?

b) Write this parametric equation in vector form **R**(t).
Find $\lim_{t \to \infty} \mathbf{R}(t)$

c) Write this parametric equation in Cartesian form.

d) What point on the Cartesian function is **not** on the parametric function?

4. Given the vector-valued function **R**(t) = $\ln(t + 1)\mathbf{i} + \tan^{-1}(t)\mathbf{j}$.

a) Give the domain of this vector-valued function.

b) Draw the graph that the tips of the vectors traverse for this vector valued function.

c) Write this vector function in parametric form.

5. Vector and parametric equations are often used for modeling the motion of an object. Set up parametric and vector equations that represent the motion described. Be sure to indicate the domain.

a) A rocket with initial velocity of 2500 feet per second is fired at an angle of 32 degrees as measured from the horizontal.

b) A player hits a baseball at an angle of 40 degrees as measured from the horizontal. The initial velocity of the baseball is 80 feet per second, and the bat is 4 feet off the ground when it hits the ball. In addition, the wind blows *against* the ball at a speed of 8 feet per second.

c) Point P is a point on the circumference of a circle of radius a. Consider the curve traced by P as the circle rolls along a straight line. Let the fixed straight line on which the circle rolls be the x-axis, and let the origin be one of the locations of P. Let t radians be the measure of the angle through which the circle rolls. (This curve is called a cycloid.)