AP Calculus BC
Lesson 12.1 Sequences

1. A sequence is a function whose domain is the set of nonnegative integers or the set of positive integers. Generate the first 20 terms of each of the following sequences using your calculator. Graph each of these sequences.

a. $a_n = 1 - \frac{1}{n}$

b. $a_1 = 1$ and $a_n = \frac{1}{2} a_{n-1}$

c. $a_n = \frac{n-1}{n}$

d. $a_n = \frac{\sin(n)}{n}$

e. $a_1 = 4$ and $a_n = 7a_{n-1} + 2$

f. $\left\{ \frac{(-1)^{n-1}(2n-3)}{n} \right\}$

2. A sequence $\{a_n\}$ converges means that $\lim_{n\to\infty} a_n = L$, where L is a constant. Otherwise, the sequence is said to diverge. Determine whether the following sequences converge or diverge. If it converges, find its limit.

a. $a_n = \dfrac{(-1)^{n+1}}{n}$

b. $a_n = \left(1 + \dfrac{1}{n}\right)^n$

c. $a_n = \dfrac{\sin(n)}{n}$

d. $a_n = \dfrac{2^n}{5n}$

e. $a_n = \dfrac{3n-1}{2n+4}$

f. $a_1 = 10$ and $a_n = \dfrac{1}{2} a_{n-1}$

g. $a_n = (-1)^n \dfrac{n-1}{n}$

h. $a_n = \left(\dfrac{2}{3}\right)^n$

i. $\begin{cases} a_1 = \sqrt{2} \\ a_n = \sqrt{a_{n-1}} \end{cases}$

j. $a_1 = .5$ and $a_n = \cos(a_{n-1})$

k. $a_1 = 1$ and $a_n = \dfrac{a_{n-1} + 3/a_{n-1}}{2}$

l. $a_n = \int_0^n e^{-x} dx$

3. Let x be a fixed number. Find the limits of the following sequences.

a. $a_n = \frac{\ln(n)}{n}$

b. $a(n) = x^{1/n}$

c. $a_n = \left(1 + \frac{x}{n}\right)^n$

d. $\left\{\sqrt[n]{n}\right\}$

e. $a_n = x^n$

f. $a_n = \frac{x^n}{n!}$

4. Let $a_n = \frac{4^n}{n!}$.

(a) Find a number N such that $a_{n+1} \leq a_n$ for all $n \geq N$.

(b) Use part (a) to explain why $\lim_{n\to\infty} a_n$ exists.

(c) Evaluate $\lim_{n\to\infty} a_n$.