AP Calculus BC
Lesson 12.1 day 2 – sequences

1. Graph the first 20 terms of the sequence using sequence mode. If the sequence converges, find the limit $\lim_{n\to\infty} a_n$.

a) $\begin{cases} a_1 = 1 \\ a_n = \frac{2}{3}a_{n-1} \text{ for } n > 1 \end{cases}$

b) $\begin{cases} a_1 = 2 \\ a_n = \frac{3}{2}a_{n-1} \text{ for } n > 1 \end{cases}$

c) $\begin{cases} a_1 = a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} \text{ for } n > 2 \end{cases}$

d) $\left\{ \frac{(-1)^{n-1}(2n-3)}{n} \right\}$

e) $a_1 = 1$ and $a_n = \frac{a_{n-1} + 3/a_{n-1}}{2}$

2. If a sequence $\{a_n\}$ is monotonic and is bounded below by 4 and above by 9, does the sequence converge? If it does converge, what do you know about the limit $\lim_{n\to\infty} a_n$?

3. Determine if the sequence is monotonic and if it is bounded:

a) $a_n = \frac{5^n}{n!}$

b) $a_n = 2n + 1$

c) $a_n = \frac{2n+1}{5n-4}$

4. Investigate the behavior of the sequence given by:

$$a_{n+1} = \begin{cases} \frac{1}{2}a_n & \text{if } a_n \text{ is an even number} \\ 3a_n + 1 & \text{if } a_n \text{ is an odd number} \end{cases}$$

if $a_1 = 11$. Try again with $a_1 = 25$. What do you think happens in the long run?

5. Find an explicit formula for the general term $\{a_n\}$, assuming that the pattern continues:

a) $\{1, 4, 7, 10, 13, ...\}$

b) $\{2, -6, 18, -54, 162, -486, ...\}$

c) $\{6, 18, 72, 360, 2160, ...\}$

d) $\left\{\frac{-2}{3}, \frac{4}{5}, \frac{-8}{7}, \frac{16}{9}, \frac{-32}{11}, ...\right\}$

6. Find value of the following expressions:

a) $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$

b) $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+...}}}$

7. Let $\{f_n\}$ be the Fibonacci numbers given by $\begin{cases} f_1 = f_2 = 1 \\ f_n = f_{n-1} + f_{n-2} \text{ for } n > 2 \end{cases}$.

a) Let $a_n = \frac{f_{n+1}}{f_n}$ and show that $a_{n-1} = 1 + \frac{1}{a_{n-2}}$.

b) Assuming that $\{a_n\}$ is convergent, find its limit.