AP Calculus BC
Lesson 12.2 Series

1. Consider the series $\sum_{k=1}^{\infty} \frac{1}{3^k}$.

(a) Explain what your calculator gives if you graph $u1 = \sum_{k=1}^{n} \frac{1}{3^k}$.

(b) How many terms must you add for the sum to be greater than .49?

(c) What is the sum of the first 100 terms of this series?

(d) Find an explicit formula for $S_n = \sum_{k=1}^{n} \frac{1}{3^k}$.

(e) If $R_n = \sum_{k=n+1}^{\infty} \frac{1}{3^k}$, find $R_{100}$.

(f) Does the series $\sum_{k=1}^{\infty} \frac{1}{3^k}$ converge or diverge? Explain your reasoning.

(g) Find an explicit formula for $R_n$.

(h) Show that $\{R_n\}$ is decreasing and bounded below.

(i) Find $\lim_{n\to\infty} R_n$.

2. Determine whether each of the following series converges or diverges:

(a) $\sum_{n=1}^{\infty} 6 \cdot \left(\frac{-1}{3}\right)^n$

(b) $\sum_{n=1}^{\infty} \frac{2}{5^n}$

3. Consider the series $S = \sum_{k=1}^{\infty} \frac{5}{(3k+1)(3k-2)}$.

(a) Find the first twenty terms of the sequence $\{S_n\}$, where $S_n = \sum_{k=1}^{n} \frac{5}{(3k+1)(3k-2)}$.

(b) Graph the first twenty terms of $\{S_n\}$.

(c) Find $\lim_{n\to\infty} S_n$.

(d) Explain how to find $S = \lim_{n\to\infty} S_n$ analytically.

3. Consider the sequence defined by $a_k = \frac{1}{k^2}$ and its sequence of partial sums $u_n$ defined by $S_n = \sum_{k=1}^{n} \frac{1}{k^2}$.

(a) Find $\sum_{k=1}^{50} \frac{1}{k^2}$.

(b) Graph the sequence of partial sums and trace the values.

(c) Graph the horizontal line $y = \frac{\pi^2}{6}$. You will need to use the **DRAW** menu.

Note: Euler proved that $\sum_{n=1}^{\infty} \frac{1}{n^{2k}} = a\pi^{2k}$, where $k$ is a positive integer and $a$ is a rational number. It can be shown that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$, and $\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$, for example.

(d) Redo problems (a) and (b) with each of the following sequences and their associated series.

(i) $u_n = \frac{1}{n}$ (ii) $u_n = (-1)^{n+1} \frac{1}{n}$ (iii) $u_n = \left(1 - \frac{1}{n}\right)^n$

5. (a) Graph the first 50 terms of the sequence of partial sums determined by the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(b) Draw the graphs of $y = \ln(x)$ and $y = \ln(x) + 1$ on the same axes. Does $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge? Explain your reasoning.

6. (a) Find $\lim_{n\to\infty} \frac{2n+1}{3n+2}$.

(b) Determine whether $\sum_{n=1}^{\infty} \frac{2n+1}{3n+2}$ converges or diverges. Explain your reasoning.

(c) Find $\lim_{n\to\infty} \frac{5}{3^n}$.

(d) Determine whether $\sum_{n=1}^{\infty} \frac{5}{3^n}$ converges or diverges. Explain your reasoning.

(e) Use the results of problems (a)-(d) to make a conjecture about $\lim_{n\to\infty} a_n$ and the convergence or divergence of $\sum_{n=1}^{\infty} a_n$.

7. If the series converges, find its sum. If it diverges, explain why.

1. $\sum_{n=1}^{\infty} \frac{2}{5^{n-1}}$

2. $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n}$

3. $\sum_{n=1}^{\infty} \frac{2^n - 1}{3^n}$

4. $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

5. $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$

6. $\sum_{n=1}^{\infty} \cos(\pi n)$

7. $\sum_{n=1}^{\infty} \left(\tan\left(\frac{\pi}{6}\right)\right)^n$

8. $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$

9. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{5}{8^n}$

10. $\sum_{n=1}^{\infty} n$

11. $\sum_{n=1}^{\infty} e^{-n}$

12. $\sum_{n=1}^{\infty} \frac{n!}{50000^n}$

13. $\sum_{n=1}^{\infty} \frac{1}{2^n + 1}$

14. $\sum_{n=0}^{\infty} \frac{1}{n!}$