AP Calculus BC
Lesson 12.3 The Integral Test

1. (a) Show that $\frac{1}{n^2} \le \int_{n-1}^{n} \frac{dx}{x^2}$.

(b) Show that $0 \le \sum_{n=2}^{\infty} \frac{1}{n^2} \le \int_{1}^{\infty} \frac{dx}{x^2}$.

(c) Determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges or diverges. Explain your reasoning.

(d) Use the results of problems (a) and (b) to show that $\int_{1}^{\infty} \frac{dx}{x^2} \le \sum_{n=1}^{\infty} \frac{1}{n^2} \le a_1 + \int_{1}^{\infty} \frac{dx}{x^2}$.

(e) Use the result of problem (d) to approximate $\sum_{n=1}^{50} \frac{1}{n^2}$.

(f) Approximate $\sum_{n=1}^{50} \frac{1}{n^2}$ by a different method.

(g) Draw a graph of the first fifty partial sums of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(h) Draw a graph of the horizontal line $y = \frac{\pi^2}{6}$ on the same axes.

2. (a) Graph the first fifty terms of the sequence of partial sums of the series $\sum_{n=1}^{\infty}\frac{1}{n}$. Do you think this series converges or diverges? Explain your reasoning.

(b) Draw the graphs of $y = \ln(x)$ and $y = \ln(x) + 1$ on the same axes. Do you think the series $\sum_{n=1}^{\infty}\frac{1}{n}$ converges or diverges? Explain your reasoning.

(c) Explain why $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\ldots \geq 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\ldots$

(d) Determine whether $\sum_{n=1}^{\infty}\frac{1}{n}$ converges or diverges. Explain your reasoning.

(e) Does $\int_1^{\infty}\frac{dx}{x}$ converge or diverge? What is the connection to the convergence or divergence of $\sum_{n=1}^{\infty}\frac{1}{n}$?

3. Determine under what conditions the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges.

4. Determine whether each series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

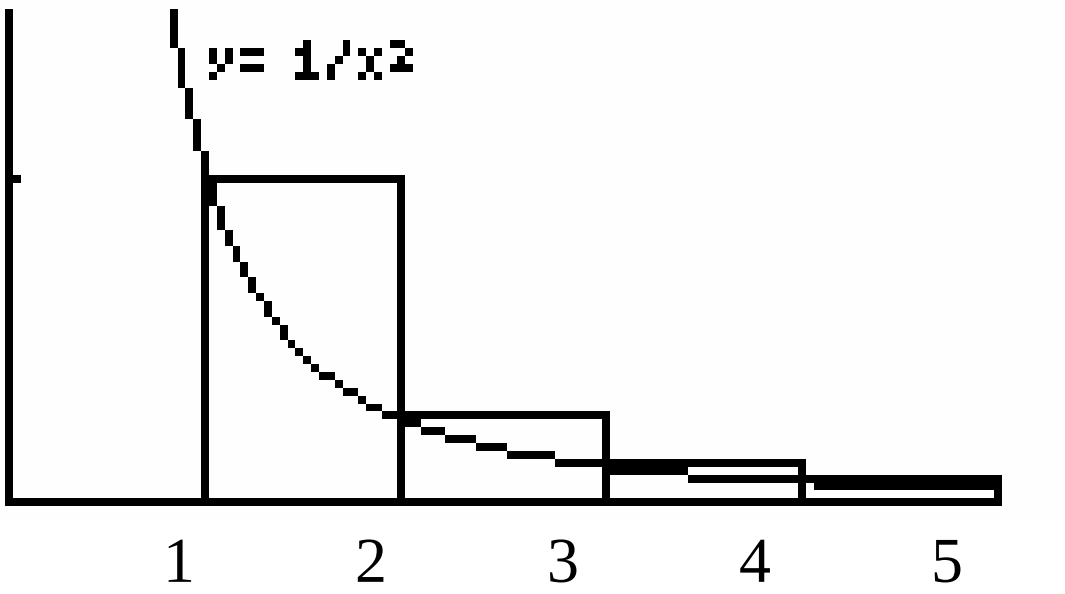
b) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

c) $\sum_{n=1}^{\infty} \frac{n^2}{4n^3+1}$

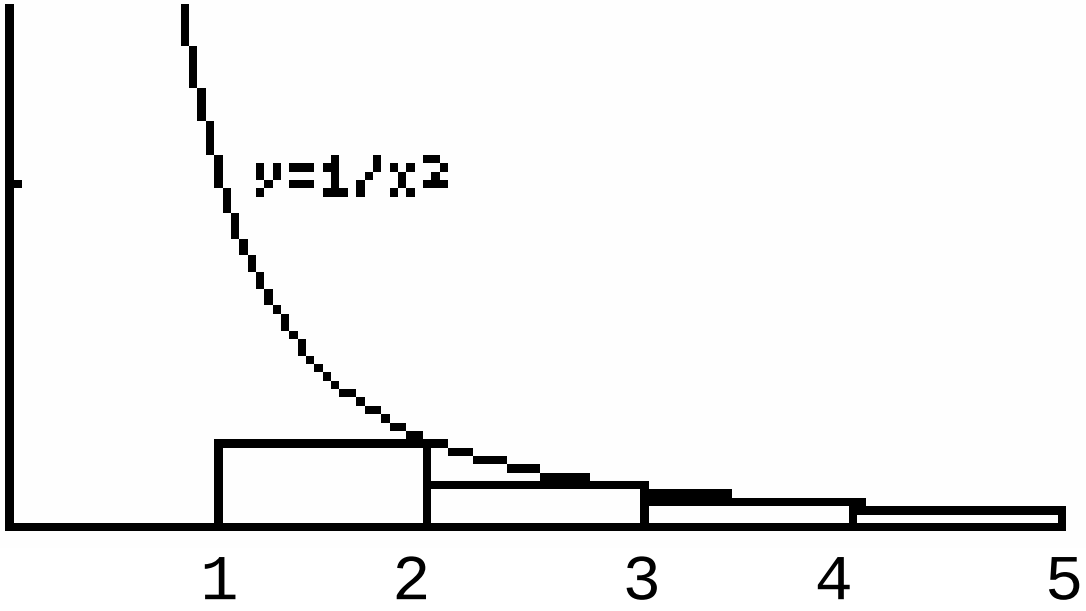
d) $\sum_{n=1}^{\infty} \frac{n}{5n^2+3}$

e) $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+4)}$

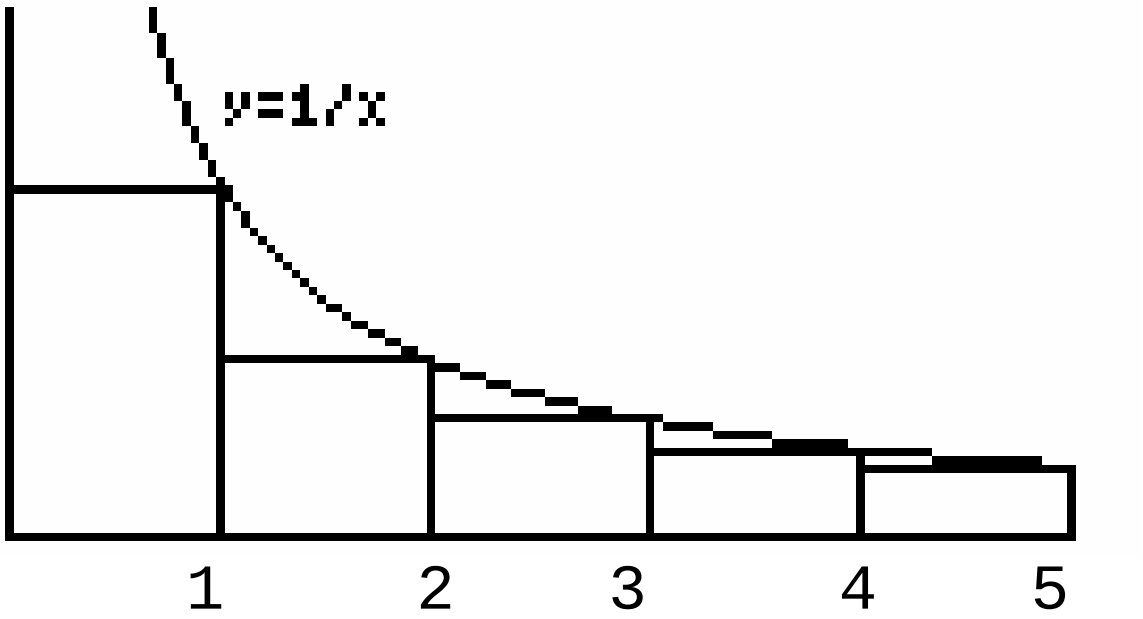
f) $\sum_{n=1}^{\infty} \frac{2n+3}{(n^2+3n)^2}$



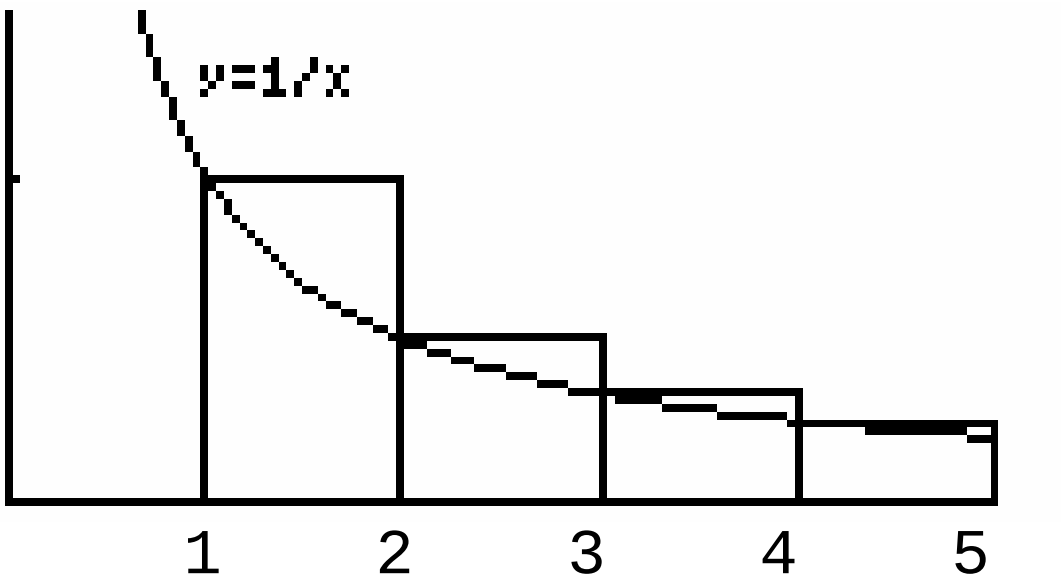
$$\sum_{n=1}^{\infty} \frac{1}{n^2} \geq \int_1^{\infty} \frac{dx}{x^2} = 1$$



$$0 \le \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \sum_{n=2}^{\infty} \frac{1}{n^2} \le 1 + \int_1^{\infty} \frac{dx}{x^2} = 2$$



$$\infty = \int_1^{\infty} \frac{dx}{x} \geq \sum_{n=1}^{\infty} \frac{1}{n} \geq 0$$



$$\sum_{n=1}^{\infty} \frac{1}{n} \geq \int_{1}^{\infty} \frac{dx}{x} = \infty$$