AP Calculus BC
Lesson 12.5 Alternating Series

1. Determine whether each series converges or diverges.

a. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n}$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n^2+1}$

c. $\sum_{n=2}^{\infty} (-1)^{n} \frac{1}{\ln(n)}$

d. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+2}$

e. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n^2}$

f. $\sum_{n=1}^{\infty} (-1)^{n} \frac{n}{2^n}$

2. Consider the alternating harmonic series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$.

(a) Compute the partial sums $S_n$ for $n = 50, 100, 200$, and 300. Are these partial sums overestimates or underestimates of the sum of the series? Explain.

(b) Compute the partial sums $S_n$ for $n = 49, 99, 199$, and 299. Are these partial sums overestimates or underestimates of the sum of the series? Explain.

(c) Show that $S_{100}$ is an underestimate with error at most 0.01 of the sum of the series.

(d) Determine an overestimate with error at most 0.01 of the sum of the series.

(e) Repeat questions (c) and (d) if the error must be at most 0.001.

(f) The limit $L$ of a convergent alternating series lies between the values of any two consecutive partial sums. This suggests using the average

$$\frac{S_n + S_{n+1}}{2} = S_n + \frac{1}{2}a_{n+1}$$

to estimate $L$. Use this to estimate $S_{100}$ and $S_{1000}$.

3. Find the sum of each series accurate to three decimal places.

a. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n!}$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n2^n}$

c. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{(2n)!}$

4. Does the infinite series $1 - \frac{1}{2^3} + \frac{1}{3^2} - \frac{1}{4^3} + \frac{1}{5^2} - \frac{1}{6^3} + \frac{1}{7^2} - \frac{1}{8^3} + \ ...$ converge or diverge? Justify your answer.