

AP Calculus BC
Lesson 12.8 Power Series

1. Consider the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$.

(a) Determine whether the series converges or diverges for:

(i) $x = \frac{1}{2}$

(ii) $x = -\frac{1}{2}$

(iii) $x = 2$

(iv) $x = -1$

(v) $x = 1$

For what values of x does the series converge? Explain your reasoning.

(b) Graph $y = \sum_{n=1}^k (-1)^{n-1} \frac{x^n}{n}$ for $k = 1, 4, 9$. What do you notice?

(c) On the same axes, graph $y = \ln(x + 1)$. What do you notice?

2. For what values of x do the following series converge?

a.
$$\sum_{n=1}^{\infty} \frac{x^n}{n+1}$$

b.
$$\sum_{n=1}^{\infty} \frac{nx^n}{3^n}$$

c.
$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

d.
$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{x^n}{n(\ln n)^2}$$

3. For each series, find the radius of convergence and the interval of convergence. Then find the derivative and the radius of convergence and the interval of convergence of the derivative.

a.
$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

b.
$$\sum_{n=2}^{\infty} \frac{(x-2)^n}{\sqrt{n-1}}$$

c.
$$\sum_{n=2}^{\infty} (-1)^n \frac{(x-3)^n}{n(n-1)}$$

4. Consider the series $\sum_{n=1}^{\infty} (2x)^n$.

(a) For what values of x does this series converge?

(b) For what values of x does this series converge absolutely?

(c) Find the sum of the series.

(d) Indicate a range of values for x for which P_{20} represents the sum with an error of at most 0.01.

5. (a) Find an infinite series for $\frac{1}{1+x^2}$. Find the interval of convergence for this series.

(b) Find an infinite series for $\tan^{-1}(x)$. Find the interval of convergence for this series.

(c) Find n such that $P_n(x)$ approximates $\tan^{-1}(x)$ with error at most 0.01.

6. (1991BC5) Let f be the function given by $f(t) = \frac{4}{1+t^2}$

and G be the function given by $G(x) = \int_0^x f(t) dt$.

(a) Find the first four nonzero terms and the general term for the power series expansion of $f(t)$ about $t = 0$.

(b) Find the first four nonzero terms and the general term for the power series expansion of $G(x)$ about $x = 0$.

(c) Find the interval of convergence of the power series in part (b). (Your solution must include an analysis that justifies your answer.)