AP Calculus BC
Lesson 12.9 Functions as Power Series

1. Find an infinite series and the interval of convergence for each of the following functions.

   a. $\dfrac{1}{1-x}$

   b. $\dfrac{1}{1+x^2}$

   c. $\tan^{-1}(x)$

2. Use your results from problem 1 to find power series for each of the following functions:

   a. $f(x) = x^3 \tan^{-1} x$

   b. $f(x) = \tan^{-1} x^2$

   c. $f(x) = \dfrac{x^2}{1-x^3}$

   d. $f(x) = \dfrac{x - \tan^{-1} x}{x^3}$

3. (a) Find a power series for $f(x) = \dfrac{1}{1+x}$.

(b) Find the interval of convergence for *f*.

(c) Find a power series for $f'(x)$.

(d) Find the interval of convergence for $f'(x)$.

(e) Find a power series for $\ln(1 + x)$.

(f) Find the interval of convergence for the series of part (e).

(g) Find a power series expansion for $\ln\left(x^2 + 1\right)$

(h) Find a power series expansion for $\ln\left(\dfrac{1+x}{1-x}\right)$.

4. Use a power series to write an infinite series of constants equivalent to each number:

   a. $\dfrac{\pi}{4}$
   (hint: use a power series for $\tan^{-1} x$)

   b. $\ln 2$
   (hint: use a power series for $\ln(x+1)$)

5. Use a power series to approximate the integral to six decimal places:

   a. $\displaystyle\int_0^{0.1} \frac{dx}{1+x^3}$

   b. $\displaystyle\int_0^{0.2} x \ln\left(1+x^2\right) dx$

6. Use a power series to evaluate each limit:

   a. $\displaystyle\lim_{x\to 0} \frac{\tan^{-1} x}{x}$

   b. $\displaystyle\lim_{x\to 0} \frac{\ln(x^3+1)}{x}$