AP Calculus BC
Lesson 2.3 Openers

1. Evaluate each of the following limits *analytically*.

a) $\lim\limits_{x\to 4}\dfrac{3x^2-8x-16}{2x^2-9x+4}$

b) $\lim\limits_{x\to -1}\dfrac{\sqrt{x+5}-2}{x+1}$

c) $\lim\limits_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$

d) $\lim\limits_{x\to 3}\dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}$

2. Find the limits: $\lim\limits_{x\to 0^+}(1+x)^{1/x}$ and $\lim\limits_{h\to 0}\dfrac{\sqrt[3]{h+1}-1}{h}$

3. Which of the following limit "rules" are true?

a) $\lim\limits_{x\to a}\left[f(x)+g(x)\right]=\lim\limits_{x\to a}f(x)+\lim\limits_{x\to a}g(x)$

b) $\lim\limits_{x\to a}\left[f(x)\cdot g(x)\right]=\left(\lim\limits_{x\to a}f(x)\right)\cdot\left(\lim\limits_{x\to a}g(x)\right)$

c) $\lim\limits_{x\to a}\left(\dfrac{f(x)}{g(x)}\right)=\dfrac{\lim\limits_{x\to a}f(x)}{\lim\limits_{x\to a}g(x)}$ if $\lim\limits_{x\to a}g(x)\neq 0$

d) $\lim\limits_{x\to a}\sqrt{f(x)}=\sqrt{\lim\limits_{x\to a}f(x)}$

4. Find the following limits.

1. $\lim_{x\to\infty} \frac{1}{x}$

2. $\lim_{x\to\infty} \frac{2x+1}{5x-2}$

3. $\lim_{x\to-\infty} \frac{7x^2-5x+4}{3x^2+4x-12}$

4. $\lim_{x\to-\infty} \frac{\sin x}{x}$

5. $\lim_{x\to-\infty} \left(3x+\frac{1}{x^3}\right)$

6. $\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)$

7. $\lim_{w\to\infty} \frac{\sqrt{w^2+4}}{w+4}$

8. $\lim_{x\to\infty} \left(3-\frac{5}{\sqrt[3]{x}}\right)\left(\tan\frac{1}{x}\right)$

5. For each of the following equations, find all vertical and horizontal asymptotes. Support your answers graphically.

1. $f(x) = \frac{2x+3}{x+7}$

2. $g(x) = \frac{4x^2}{x^2-4}$

3. $3xy-2x-4y-3=0$

4. $x^2y+4xy-x^2+x+4y-6=0$

6. Consider the function defined by $f(x) = \dfrac{x^2 - 5x + 3}{x - 2}$.

1. Draw a complete graph of $y = f(x)$. Describe the end behavior of the function.
2. Let $g(x) = x^2 - 5x + 3$ and $h(x) = x - 2$. Rewrite $f(x)$ as $q(x) + \dfrac{r(x)}{h(x)}$, where deg r < deg h.
3. Graph $y = f(x)$ and $y = q(x)$ on the window [-6,12] by [-15,15]. Predict what happens if you zoom out.
4. Find an equation of the function which describes the end behavior of $f(x)$.

7. Consider the function defined by $f(x) = \dfrac{g(x)}{h(x)}$, where $g(x) = x^3 + 8x^2 + 7x - 16$ and $h(x) = x + 3$.

1. Draw a complete graph of $y = f(x)$. Describe the end behavior of the function.
2. Rewrite $f(x)$ as $f(x) = q(x) + \dfrac{r(x)}{h(x)}$, where deg r < deg h.
3. Graph $y = f(x)$ and $y = q(x)$ on the window [-16,10] by [-25,25]. Predict what happens if you zoom out.
4. Find an equation which describes the end behavior of $f(x)$.

8. Find the end behavior asymptote and all vertical asymptotes for each of the following functions.

1. $f(x) = \dfrac{5x - 6}{3x + 9}$
2. $g(x) = \dfrac{2x^3 - 5x^2 - 3x + 5}{x - 4}$
3. $h(x) = \dfrac{2x^3 - 5x^2 - 3x + 5}{x^2 - 2x - 8}$

9. Evaluate each limit.

1. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 1} - x \right)$
2. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x} - x \right)$