AP Calculus BC
Lesson 2.4 Openers: Limits involving infinity

1. Consider the following limit proof:

$\forall \varepsilon > 0$

Let $M = \max\left(\dfrac{1}{4\varepsilon} - \dfrac{1}{2}\ ,\ 0\right)$

If $x > M$ then

a) $2x + 1 > 0$

b) $2x + 1 > \dfrac{1}{2\varepsilon}$

c) $\left|\dfrac{1}{2x+1}\right| < 2\varepsilon$

d) $\left|\dfrac{x}{2x+1} - \dfrac{1}{2}\right| < \varepsilon$

Since $\forall \varepsilon > 0$, $\exists\, M > 0$ such that $x > M \Rightarrow \left|\dfrac{x}{2x+1} - \dfrac{1}{2}\right| < \varepsilon$, $\lim\limits_{x \to \infty} \dfrac{x}{2x+1} = \dfrac{1}{2}$

Explain the steps lettered a) through d)

2. Consider the function with equation $f(x) = 1/x$.

a) Find M such that $x > M \Rightarrow \left|f(x) - 0\right| < 0.01$.

b) Find M such that $x > M \Rightarrow \left|f(x) - L\right| < \varepsilon$ $\forall\ \varepsilon > 0$.

c) What does problem 2 prove? Explain.

3. Prove each of the following.

a) $\lim_{x\to\infty} \frac{2x+5}{x-6} = 2$

b) $\lim_{x\to\infty} \frac{4x^2}{x^2+2} = 4$

c) $\lim_{x\to\infty} \frac{x^2+2x}{x^2-1} = 1$

d) $\lim_{x\to\infty} \frac{ax+b}{cx+d} = \frac{a}{c}$

e) $\lim_{x\to\infty}(3x+5) = \infty$

(Hint: Show that for any $N > 0$ there exists an $M > 0$ such that $x > M \Rightarrow 3x+5 > N$.)

4. You have been told that $\lim_{x\to 0} \frac{|x|}{x} = 0$. You challenge that statement with an $\varepsilon > 0$ for which you are certain he cannot find a $\delta > 0$. Describe a possible value of ε which you might give to him and describe why he won't be able to find a δ.