AP Calculus BC
Lesson 2.4 Openers Part 1: Limit Definition - Linears

1. If $f(x) = 3x + 4$, then $f(2) = 10$.
   How close to $x_0 = 2$ must we hold x to be sure that f lies within
   a. 2 units of $y_0 = 10$?

   b. $\frac{1}{10}$ units of $y_0 = 10$?

   Estimate graphically and confirm algebraically.

2. If $f(x) = 4x - 6$, then $f(3) = 6$.

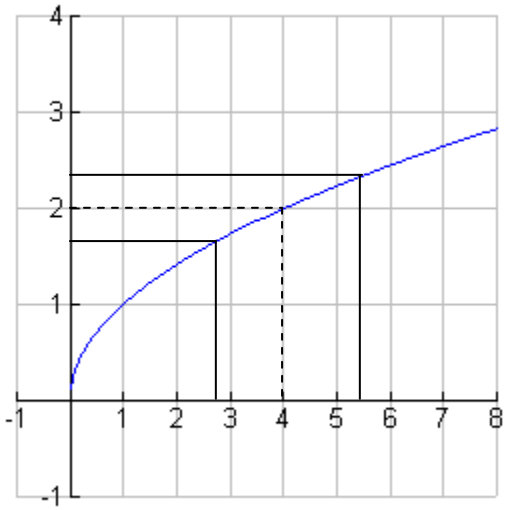
   a) Let $\varepsilon = 0.01$. Find a value $\delta > 0$ for which $|x-3| < \delta \Rightarrow |f(x)-6| < \varepsilon$.

   b) For your δ, prove that $0 < |x-3| < \delta \Rightarrow |f(x)-6| < \varepsilon$.

3. Given the function $f(x) = \sqrt{x}$ shown at right:

   Find a value for δ such that

   $|x-4| < \delta \Rightarrow |f(x)-2| < 0.3$



4. The formal definition of $\lim_{x \to a} f(x) = L$ is given as follows:

   $\forall \varepsilon > 0,\ \exists \delta > 0$ such that $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

   Explain this definition visually.

5. Use the formal $\delta - \varepsilon$ definition to prove that $\lim_{x \to 3}(-2x + 5) = -1$.