AP Calculus BC
Lesson 2.4 Openers: Limit Definition (Non-linear)

1. Following is a limit proof:

$\forall \varepsilon > 0$

Let $\delta = \min\left(\frac{\varepsilon}{2}, 1\right)$

If $0 < |x-3| < \delta$ then

a) $|x-3| < \frac{\varepsilon}{2} \Rightarrow 2|x-3| < \varepsilon$

b) $|x-3| < 1 \Rightarrow 1 > \frac{1}{|x-1|}$

c) $2 \cdot \frac{1}{|x-1|}|x-3| < 2 \cdot 1 \cdot |x-3| < \varepsilon$ and

d) $2 \cdot \frac{1}{|x-1|}|x-3| = \left|\frac{4}{x-1} - 2\right|$

So finally $\left|\frac{4}{x-1} - 2\right| < \varepsilon$

Since $\forall \varepsilon > 0$, $\exists\ \delta > 0$ such that $0 < |x-3| < \delta \Rightarrow \left|\frac{4}{x-1} - 2\right| < \varepsilon$, $\lim\limits_{x \to 3} \frac{4}{x-1} = 2$.

Explain the steps indicated in a), b), c) and d).

2. Prove each of the following using the formal definition:

a. $\lim\limits_{x \to 1} x^2 = 1$

b. $\lim_{x \to 4} \frac{1}{x} = 0.25$

c. $\lim_{x \to 2} \left(x^2 - 2x + 1 \right) = 1$

d. $\lim_{x \to 2} \left(x^2 + 2x - 1 \right) = 7$

e. $\lim_{x \to 9} \sqrt{x-5} = 2$

f. $\lim_{x \to 5} \frac{6}{2x-4} = 1$