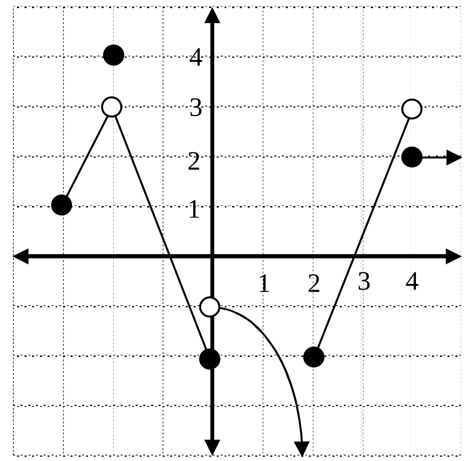
AP Calculus BC
Lesson 2.5 Continuity

1. Given the graph of f(x) shown at right, answer the following:



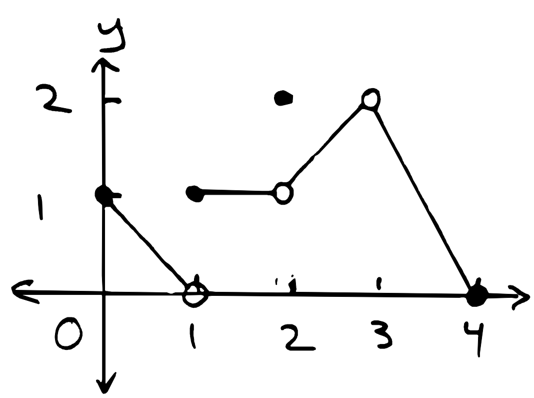
a) at what values of *x* is *f*(*x*) NOT continuous? Why?

b) at what values of *x* is *f*(*x*) continuous from the right only?

c) at what values of *x* is *f*(*x*) continuous from the left only?

d) At what values of *x* does *f*(*x*) have a removable discontinuity?

2. The graph of $y = f(x)$ is shown at right.



a) Are there any values of *x* on [0,4]

for which $f(x)$ does not exist?

b) Are there any values of *a* on [0,4] for which $\lim_{x \to a} f(x)$ does not exist?

c) Are there any values of *a* on [0,4] for which $\lim_{x \to a} f(x) \neq f(a)$?

d) For what values of *a* on [0,4] is the graph of f discontinuous?

e) For which values you found in part d) can you make f continuous?

3. Let $y = f(x)$ be defined by $f(x) = \begin{cases} 1, x < 0 \\ \sqrt{1-x^2}, 0 \le x \le 1 \\ x-1, x > 1 \end{cases}$. Determine where this function is discontinuous. Explain your reasoning.

4. What value of a will make $f(x) = \begin{cases} x^2 - 1, x < 3 \\ 2ax, x \ge 3 \end{cases}$ continuous at $x = 3$? Justify your answer.

5. f is continuous on $(-\infty, -2)$, [-2,4], and $(4, \infty)$.

(i) $\lim_{x \to -5} f(x) = 0$

(ii) $\lim_{x \to -2^-} f(x) = -\infty$

(iii) $\lim_{x \to -2^+} f(x) = -3$

(iv) $\lim_{x \to 0} f(x) = -1$

(v) $\lim_{x \to 4^-} f(x) = 2$

(vi) $\lim_{x \to 4^+} f(x) = 5$

(vii) $\lim_{x \to 6} f(x) = 0$

Draw a sketch of a possible graph of f.

6. An important theorem of any function that is continuous on a closed interval, is known as the **INTERMEDIATE VALUE THEOREM** which states that, if f is a continuous function on [a,b], then f assumes every value between f(a) and f(b)

Suppose f is a continuous function on [2,5] and f(2) = -6 and f(5) = 7 explain why there must be a zero for f somewhere in (2,5).

Suppose that f(x) = e^x on [0,ln 4]. Without graphing, explain why there must be a $p \in (0, \ln 4)$ where f(p) = 3.

Show by counterexamples why it is important that f be continuous throughout the [a,b] in order for the Intermediate Value Theorem to be true. Consider 2 cases: a single discontinuity at an endpoint, and a single discontinuity at an interior point.

7. Explain how the intermediate value theorem guarantees that there is a c, such that 1 < c < 4 and that f(c) = 7, when f(x) = $2^x + 1$.

Find the c guaranteed by the intermediate value theorem for the function in part (a). Either give an exact answer or an answer rounded to 3 decimal places.

8. Let f be defined by $f(x) = 4 + 3x - x^2$ on [2,5].
Verify that the number k = 1 is one of the numbers between f(2) and f(5). Find the number c guaranteed by the intermediate value theorem.

9. Show that the intermediate value theorem guarantees a solution to $x^3 + x + 3 = 0$ somewhere between x = -2 and x = -1.