**AP Calculus BC**
**Lesson 3.10 Linearization and Differentials**

1. The tangent line to a curve at a point on that curve is called the linearization (or linear approximation) of the function at that point. This is denoted by the function $L(x)$.

   a) Find the linearization $L(x)$ to $f(x) = \sqrt[3]{x}$ at $x = 8$.

   b) Compare $L(8)$ and $f(8)$. Compare $L'(8)$ and $f'(8)$. What do you notice?

   c) How closely does this linear equation approximate $f$ at $x = 8$? At $x = 9$?

   d) Find an interval for which the linear approximation you found in part (a) approximates $f$ to an accuracy of 0.1.

2. a) Approximate $\sqrt{37}$ by using a linear approximation to $f(x) = \sqrt{x}$ at $x = 36$. What is the error when compared to the actual value of $\sqrt{37}$ ?

   b) Rewrite your linear approximation in the form $l(x) = a + b(x - 36)$. What are the values of $a$ and $b$?

3. Find $\Delta f$, $df$, the error $|\Delta f - df|$, the relative change, and the percentage change for each of the following:

   a. $f(x) = 2x^2 + 4x - 3$, when $x = 1$ and $dx = 0.5$

   b. $f(x) = \sin(x)$, when $x = \frac{\pi}{6}$ and $dx = \frac{\pi}{180}$

   c. $f(x) = \frac{1}{x}$, when $x = 2$ and $dx = -0.1$

4. Write a differential formula that estimates the given change in volume or surface area for each of the following situations.

   a. The change in the volume $v = x^3$ of a cube when the edge of the cube changes from $x_0$ to $x_0 + dx$.

   b. The change in the lateral surface area $S = 2\pi rh$ of a right circular cylinder when the height changes from $h_0$ to $h_0 + dh$.

5. The edge of a cube is measured as 10 cm with an error of 1%. The volume of the cube is calculated from this measurement. Estimate the percentage error in the calculation of the volume.

6. Estimate the volume of rubber in a basketball if the inner diameter is 14 inches and the outer diameter is 14.25 inches.

7. A manufacturer contracts to mint coins for the government. How much variation in $dr$, where $r$ is the radius of the coin, can be tolerated if the coins are to weigh within $\frac{1}{1000}$ of their ideal weight? Assume that the thickness does not vary.

8. Estimate the allowable percentage error in measuring the diameter $d$ of a sphere if the volume is to be calculated to within 3%.