AP Calculus BC
Lesson 3.3

There are many notations for indicating the derivative of a function at a point or the derivative function of a function.

<u>Derivative of a function at a point</u>

$\left.\frac{dy}{dx}\right|_{x=a}$ or $f'(a)$ or $D_y(a)$ or $\dot{y}(a)$

<u>Derivative function</u>

$\frac{dy}{dx}$ or $f'(x)$ or $D_y(x)$ or $\dot{y}$

The first notation in each column is called Leibnitz notation after one of the two founders of Calculus. The last notation in each column is called Newton notation after the other founder of Calculus.

1. Use the definition of a derivative to prove the following:

a) $\frac{d(x^3)}{dx} = 3x^2$

b) $\frac{d(x^n)}{dx} = nx^{n-1}$ for $n \in$ {positive integers}

c) $\frac{d(c)}{dx} = 0$ for any constant c

d) $\frac{d(cf(x))}{dx} = c\frac{d(f(x))}{dx}$

2. Prove $\frac{d(x^{-2})}{dx}$ = $-2x^{-3}$

Prove $\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

What general statement could you make about $\frac{d(x^n)}{dx}$, where n is rational or negative?

Additional theorems to know:

Given that *u* and *v* are differentiable functions of *x*, then

a) $\frac{d(u+v)}{dx} = \frac{d(u)}{dx} + \frac{d(v)}{dx}$

b) $\frac{d(u-v)}{dx} = \frac{d(u)}{dx} - \frac{d(v)}{dx}$

3. For each of the following functions, find $f'(x)$.

a) $f(x) = x^4 - 7x^3 + 2x^2 + 15$

b) $f(x) = \frac{x^4}{4} - \frac{3x^2}{2} - x$

c) $f(x) = (x^2 - 4x + 5)(3 - x^3)$

d) $f(x) = \frac{5}{6x^2}$

e) $f(x) = \frac{12}{x} - \frac{4}{x^3} + \frac{1}{x^4} - 7$

4. For each of the following functions, find a function $F(x)$ such that $F'(x) = f(x)$.
This function $F(x)$ is called the *antiderivative* of function $f(x)$.

a) $f(x) = x^4 - 7x^3 + 2x^2 + 15$

b) $f(x) = \frac{x^4}{4} - \frac{3x^2}{2} - x$

c) $f(x) = (x^2 - 4x + 5)(3 - x^3)$

d) $f(x) = \frac{5}{6x^2}$

e) $f(x) = \frac{12}{x} - \frac{4}{x^3} + \frac{1}{x^4} - 7$

5. Suppose f and g are functions related by the equation $g(x) = f(x+3)\ \forall x$.

a) Explain, with words and pictures, why f' and g' are then related by $g'(x) = f'(x+3)$.

b) Use part a) to compute $g'(x)$ if $g(x) = \dfrac{1}{(x+3)^4}$.

6. Given $f(x) = x^3 - 9x$.

a) Write an equation of the tangent line to the graph of this function where $x = 3$.

b) Write an equation of the tangent line to the graph of this function that also passes through the point (3,0).