AP Calculus BC
Lesson 3.3 continued

Suppose that u and v are differentiable functions of x. Many beginning calculus students believe that $\frac{d}{dx}(uv) = \frac{du}{dx} \cdot \frac{dv}{dx}$. Unfortunately for them, this is <u>not</u> a true statement.

1. Consider the functions $u = x^2 + 1$ and $v = x^4 - x^2 + 1$.

    (a) Calculate $\frac{du}{dx} \cdot \frac{dv}{dx}$.

    (b) Calculate uv.

    (c) Calculate $\frac{d}{dx}(uv)$.

2. Explain why if the statement $\frac{d}{dx}(uv) = \frac{du}{dx} \cdot \frac{dv}{dx}$ were true, the derivative of every function would be zero.

3. Find $\frac{dy}{dx}$ for the function $y = (x^3 + 2x)(4x^2 - 5)$

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4. Prove $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$. *The Product Rule*

5. For each of the following functions, find $f'(x)$.

1. $f(x) = (x^2 - 4x + 5)(3 - x^3)$

2. $f(x) = (x^2 + 1)(x^3 + 3)$

3. $f(x) = (x - 2)(x^2 - 7x)(x^2 - 4)$

4. $f(x) = \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

5. $f(x) = \frac{(x-1)(x^2 + 5x)}{x^3}$

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Suppose that u and v are differentiable functions of x. Many beginning calculus students believe that $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{du/dx}{dv/dx}$. Unfortunately for them, this is a false statement.

6. Consider the functions $u=2x+1$ and $v=x-1$.

(a) Calculate $\dfrac{\frac{du}{dx}}{\frac{dv}{dx}}$.

(b) Use the limit definition to calculate $\frac{d}{dx}\left(\frac{u}{v}\right)$

7. Explain why if the statement $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{du/dx}{dv/dx}$ were true, the derivative of every function would be undefined.

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8. Prove $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$. *The Quotient Rule*

9. Find the derivative for each of the following functions.

1. $y=\frac{x^5-3x^3+1}{4}$

2. $f(x)=\frac{x}{x+1}$

3. $f(x)=\frac{x}{2\sqrt{x}+3}$

4. $y=\frac{x^2-5x+2}{3x+1}$

5. $f(x)=\frac{5}{6x^2}$

6. $f(x)=\frac{12x^3-4x+1-7x^4}{x^4}$

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10. Using the Quotient Rule, prove $\frac{d(x^{-4})}{dx} = -4x^{-5}$.

11. Using the Quotient Rule, prove $\frac{d(x^{n})}{dx} = nx^{n-1}$ if n is a negative integer.

12. Suppose that u and v are functions that are differentiable at $x = 2$ and that $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$.
Find the values of the following derivatives at $x = 2$.

1. $\frac{d}{dx}(uv)$
2. $\frac{d}{dx}\left(\frac{u}{v}\right)$
3. $\frac{d}{dx}\left(\frac{v}{u}\right)$
4. $\frac{d}{dx}(3u - 2v + 2uv)$

13. Find a general antiderivative for each of the following functions:

a. $f(x) = (2x^2 + 3x + 1)(3x^2 - 8x) + (4x + 3)(x^3 - 4x^2 - 5)$

b. $f(x) = \frac{(x-3)(2x) - (x^2 + 1)}{(x-3)^2}$