AP Calculus BC
Lesson 3.5 Derivatives of Trig Functions

1. Consider the function $y = \sin(x)$. Use your calculator to draw $\frac{d}{dx}\left(\sin(x)\right)$.

   According to your evidence, $\frac{d}{dx}\left(\sin(x)\right) = ?$

2. Consider the function $y = \cos(x)$. Use your calculator to draw $\frac{d}{dx}\left(\cos(x)\right)$.

   According to your evidence, $\frac{d}{dx}\left(\cos(x)\right) = ?$

3. Consider the limits $\lim_{h\to 0}\frac{\sin(h)}{h}$ and $\lim_{h\to 0}\frac{\cos(h)-1}{h}$

   a) Identify each limit as a derivative $f'(a)$. What is $f(x)$ and what is a?

   b) Find the values for these limits by looking at the graph of the function $f(x)$.

   c) Verify the value of the limits using the **limit** feature on your calculator.

4. Prove, using the limit definition of the derivative, that:

   a. $\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$

   b. $\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$

5. Find $\frac{dy}{dx}$ for each of the following functions.

   a. $y = 1 + x - \cos(x)$

   b. $y = \frac{1}{x} + 5 \cdot \sin(x)$

   c. $y = x^2 - \sin(x)$

   d. $y = \sin(x) \cdot \cos(x)$

6. Find equations for the lines that are tangent and normal to the curve $y = \sqrt{2}\cos(x)$ at the point $(\pi/4, 1)$.

7. A weight hanging from a string is compressed 5 units above its rest position ($s = 0$) and released at time $t = 0$ to bob up and down. Its position at any later time t is $x = 5 \cdot \cos(t)$. What are its velocity and acceleration at time t?

8. Write equations of the tangent line and the normal line to the curve $f(x) = \tan(x)$ where $x = \pi/3$.

9. Use the Quotient Rule to prove each of the following.

a. $$\frac{d\left(\tan(x)\right)}{dx} = \sec^2\left(x\right)$$

b. $$\frac{d\left(\cot(x)\right)}{dx} = -\csc^2\left(x\right)$$

c. $$\frac{d\left(\sec(x)\right)}{dx} = \sec\left(x\right)\tan\left(x\right)$$

d. $$\frac{d\left(\csc(x)\right)}{dx} = -\csc\left(x\right)\cot\left(x\right)$$

10. Find $\frac{dy}{dx}$ for each of the following.

a. $y = 2\cos(x) - \tan(x)$

b. $y = x^2 \cot(x)$

c. $y = \sin(x)\csc(x)$

d. $y = \sin(x)\cos(x)$

e. $y = \frac{\sin(x)}{1+\cos(x)}$

f. $y = \frac{1+\tan^2(x)}{\cot(x)}$

11. (1984AB2) Let *f* be the function defined by $f(x) = \frac{x+\sin(x)}{\cos(x)}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

a. State whether *f* is an even function or an odd function. Justify your answer.

b. Find $f'(x)$.

c. Write an equation of the line tangent to the graph of *f* at the point where $x = 0$.

12. (1985AB2BC1) A particle moves along the x-axis with acceleration given by $a(t) = \cos(t)$ for $t \geq 0$. At $t = 0$ the velocity $v(t)$ of the particle is 2 and the position $x(t)$ is 5.

a. Write an expression for the velocity $v(t)$ of the particle.

b. Write an expression for the position $x(t)$.

c. For what values of t is the particle moving to the right? Justify your answer.

d. Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.