AP Calculus BC
Lesson 3.6 The Chain Rule

3.6(1)

1. Graph the function $f(x) = \sin(3x)$.
2. Graph what you believe to be $f'(x)$. Be brave, make a guess!
3. Graph the derivative of $f(x)$.
4. If your guess was correct explain why. If it was not (after doing problem 3) make another guess and see if you are now correct.

3.6(2)

1. Graph $f(x) = \dfrac{x^3 - x^2 + x}{2}$ and $y = f(2x)$.
2. Find an equation of the tangent to $f(x)$ at (1,0.5) and the tangent to $f(2x)$ at (0.5,0.5). What is the relationship between the slope on $f(x)$ and $f(2x)$ at corresponding points? Explain.

3.6(3)

1. Consider $f(x) = (3x + 1)^2$. Calculate $\dfrac{d}{dx}\left(f\left(x\right)\right)$.
2. Compare your answer to $f'(x) = 2(3x+1)\cdot 3$
3. What is the effect of the 3 in front of the x on the graph of f(x), and what influence does this have on the slope?

3.6(4)

1. Consider $f(x) = \tan^2 x$. Calculate $\dfrac{d}{dx}\left(f\left(x\right)\right)$ using the product rule.
2. Is the answer equivalent to $2\tan x$ or $\left(\sec^2 x\right)^2$? Why or why not?

3.6(5)

1. Consider $f(x) = \tan^3 x$. Calculate $\dfrac{d}{dx}\left(f\left(x\right)\right)$ using the product rule.
2. Is the answer equivalent to $3\tan^2 x$ or $\left(\sec^2 x\right)^3$? Why or why not?

The Chain Rule:

If $h(x) = f\left(g(x)\right)$, then $h'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dh}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

3.6(6) Find $f'(x)$ or $\frac{dy}{dx}$ for each of the following.

1. $f(x) = (x^2 - 4x)^5$

2. $f(x) = \sin(\tan(x))$

3. $f(x) = \sqrt{x + \sin(x)}$

4. $f(x) = (x^2 - 4x)^5 (2x - 1)^3$

5. $f(x) = \sin^2(3x^2 - 5)$

6. $y = x \cdot \sec(4x^2)$

7. $y = \csc^5(4x^2)$

3.6(7)

Find an antiderivative for each of the following functions:

1. $f(x) = 3\left(x^2 - 5x + 7\right)^2 \left(2x - 5\right)$

2. $f(x) = 6x^3 \sin\left(3x^4 + 2\right)$

3. $f(x) = 12x^2 \tan^3\left(x^3\right) \sec^2\left(x^3\right)$

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3.6(8)

If $u = w^3 - 4w$ and w = sin(4v) and v = $x^2 - \frac{1}{x}$, find

1. *du*/*dw* 2. *du*/*dv* 3. *du*/*dx* 4. *dw*/*dx*

3.6(9)

If $f(x) = (4x^2 + 4x + 1)^3$, compute

1. $\frac{d(f(x))}{d(4x^2 + 4x + 1)}$ 2. $\frac{d(f(x))}{d(2x)}$

3. $\frac{d(f(x))}{d(2x + 1)}$ 4. $\frac{d(f(x))}{dx}$

3.6(10)

Prove each of the following using the chain rule.

1. The derivative of an odd function is an even function.

2. The derivative of an even function is an odd function.