AP Calculus BC
Lesson 3.7 Implicit Differentiation

3.7(1) Find the following, assuming that *y* is defined implicitly as a function of *x*.

1. $\dfrac{d(x^2)}{dx}$

2. $\dfrac{d\left(\left(y(x)\right)^2\right)}{dx}$

3. $\dfrac{d\left(x\cdot y(x)\right)}{dx}$

4. $\dfrac{d(x^2\sin(y(x)))}{dx}$

5. $\dfrac{d\left(\sin\left(x\cdot y(x)\right)\right)}{dx}$

3.7(2) Given $16x^2+9y^2=144$.

1. Graph this relation using your graphing calculator.

2. Find *dy*/*dx* for each functional portion of this relation at any value of *x* and also at *x* = 2.

3. Take the original relation and differentiate both sides with respect to *x* and then solve for *dy*/*dx*. Compare your result to your results in number 2.

4. Substitute $x=2$ and $y=\dfrac{4\sqrt{5}}{3}$ into your equation in number 3. Substitute $x=2$ and $y=\dfrac{-4\sqrt{5}}{3}$ into your equation for number 3.

5. Compare your results from numbers 2 and 4.

3.7(3) Find *dy*/*dx* using implicit differentiation for each of the following:

1. $x^2 + y^2 = 8$

2. $x^2 - 4y^2 = 1$

3. $x + \sin(y) = y^2$

4. $2xy + y^2 = x + y$

5. $x^2 + \sin(xy) = 3x^2 y$

3.7(4)

1. Prove that $\frac{d(x^{\frac{3}{4}})}{dx} = \frac{3}{4} x^{\frac{-1}{4}}$ by doing the following:
Let $y = x^{3/4}$ and then take the fourth power of each side. Final find *dy*/*dx* by implicit differentiation.

2. Prove that $\frac{d(x^{\frac{p}{q}})}{dx} = \frac{p}{q} x^{\frac{p}{q}-1}$ by using the same technique as in number 1.

3.7(5) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of the following:

1. $x^2 - 4y^2 = 1$

2. $xy + y^2 = 1$

3.7(6) (1978AB5BC1) Given the curve $x^2 - xy + y^2 = 9$.

1. Write a general expression for the slope of the curve.

2. Find the coordinates of the points on the curve where the tangents are vertical.

3. At the point (0,3) find the rate of change in the slope of the curve with respect to x.

3.7(7) (1987BC2) Consider the curve given by the equation $y^3 + 3x^2y + 13 = 0$.

1. Find $\frac{dy}{dx}$.

2. Write an equation for the line tangent to the curve at the point (2,-1).

3. Find the minimum y-coordinate of any point on the curve. Justify your answer.

3.7(8)

(1980AB6BC4) Let $y = f(x)$ be the continuous function that satisfies the equation $x^4 - 5x^2y^2 + 4y^4 = 0$ and whose graph contains the points (2,1) and (-2,-2). Let L be the line tangent to the graph of f at $x = 2$.

1. Find an expression for y'.

2. Write an expression for line L.

3. Give the coordinates of a point that is on the graph of f but is not on line L.

4. Give the coordinates of a point that is on line L but is not on the graph of f.