BC Calculus
Lesson 3.9 Related Rates

3.9(1) If $xy = 10$ and $\frac{dx}{dt} = -20$, find $\frac{dy}{dt}$ when $x = -5$.

3.9(2)

(Dedicated to Mr. Ray – a math teacher who once actually had the ladder fall… He broke his arm but had a great problem for his classes…)

A man is standing atop a 20-foot ladder that is leaning against a wall. Suddenly the base of the ladder moves away from the wall at a constant rate of 2 ft/sec. At what rate is the man falling when the base of the ladder is 5 ft from the wall?

3.9(3)

A baseball diamond is a square 90 feet on a side. A runner starts from home plate towards first base at 20 ft/sec. How fast is the runner's distance from second base changing when the runner is halfway to first base? Is this distance increasing or decreasing? Why?

3.9(4)

An ice cream cone with an altitude of 15 cm and base radius of 4 cm is initially filled with melted vanilla ice cream. A small hole develops at the bottom of the cone, and the ice cream "flows" out at a rate of 20 cm^3/min. How fast is the height of the melted ice cream dropping when the height of the melted ice cream is 5 cm?

3.9(5)

1990AB4

The radius *r* of a sphere is increasing at a rate of 0.04 centimeters per second. (Note: The volume of a sphere with radius *r* is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

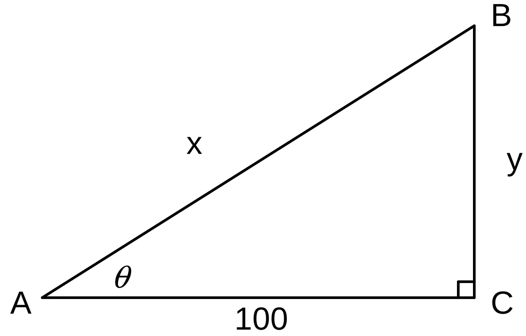
(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

3.9(6)

1988BC3

The figure below represents an observer at point *A* watching balloon *B* as it rises from point *C*. The balloon is rising at a constant rate of 3 meters per second, and the observer is 100 meters from point *C*.



(a) Find the rate of change in *x* at the instant when $y = 50$.

(b) Find the rate of change in the area of right triangle *BCA* at the instant when $y = 50$.

(c) Find the rate of change in θ at the instant when $y = 50$.

3.9(7)

[1984 AB5, BC2] A balloon is in the shape of a cylinder and has hemispherical ends of the same radius as that of the cylinder. (i.e., it looks like a medicine capsule). The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant that the radius of the cylinder is 3 cm, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.

a) At this instant, what is the height of the cylinder?
b) At this instant, how fast is the height of the cylinder changing?

3.9(8)

[1987 AB5] The trough has an inverted isosceles triangle as a base. This isosceles triangle has a base of 2 feet and a height of 3 feet. The trough is 5 feet long. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let H be the depth and V be the volume of water in the trough.

a) Find the volume of water in the trough when it is full.

b) What is the rate of change in H at the instant when the trough is ¼ full by volume?

c) What is the rate of change in the area of the surface of the water, at the instant when the trough is ¼ full by volume?