AP Calculus BC - Practice with Optimization Problems

1. A tank with a rectangular base and rectangular sides is open at the top. It is constructed so that the width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank? Justify your answer.

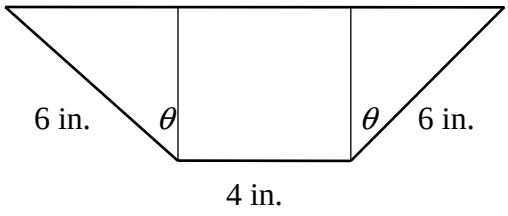
2. Let $f(x) = 6 - x^2$. For $0 < w < \sqrt{6}$, let $A(w)$ be that area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(w,\ 6 - w^2)$.

   a) Find $A(1)$.
   b) For what value of w is $A(w)$ a minimum?

3. The US postal service will accept a box for domestic shipment only if the sum of the length and the girth, distance around, does not exceed 108 inches. Find the dimensions of the largest acceptable box with a square end. Justify your answer.

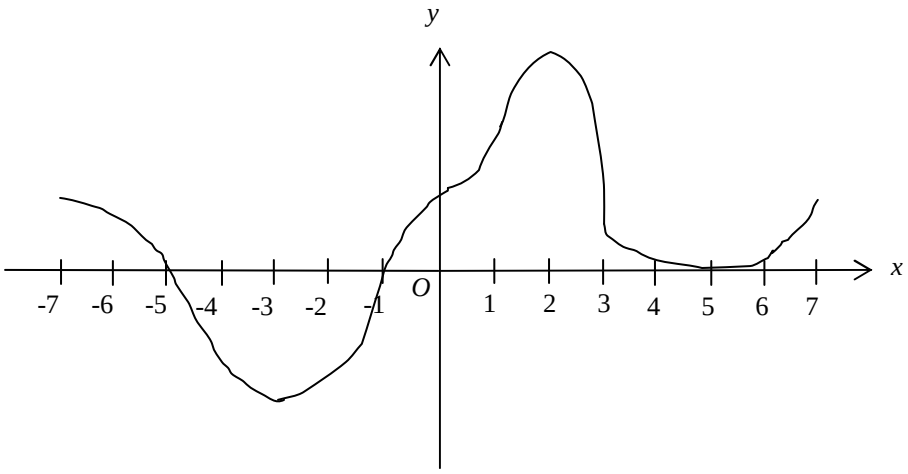
4. Find the volume of the largest right circular cone that can be inscribed inside a sphere of radius 3. Justify your answer.

5. A gutter is to be made whose end in the shape of an isosceles trapezoid shown below



What will be the value of θ when the gutter holds the most water? Justify your answer.

6.



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

c) Find all values of x, for $-7 < x < 7$, at which $f'(x) < 0$.

d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.