AP Calculus BC
Lesson 4.10 Antiderivatives

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

In general, you should give <u>the most general</u> antiderivative possible.

1. Find an antiderivative for each function given:

   a. $f(x) = 4x^3 + 2x - 7$

   b. $f(x) = \sqrt{x} - \dfrac{1}{x^2}$

   c. $f(x) = 3\cos x - 4\sin x$

   d. $f(x) = \sec^2 x - \csc x \cot x$

   e. $f(x) = \sqrt[5]{x} - 3x^4 + \dfrac{4}{x^{5/3}} - 5\csc^2 x$

2. Find an antiderivative for each function given:

   a. $f(x) = \dfrac{2x\tan x - x^2 \sec^2 x}{\tan^2 x}$

   b. $f(x) = 3x^2 \sin x + x^3 \cos x$

   c. $f(x) = 3\sin(4x) - \cos(2x) + \sec(3x)\tan(3x) + 7$

   d. $f(x) = 9\left(3x^4 - 4x\right)^8 \left(12x^3 - 4\right)$

   e. $f(x) = 8x\sin^3\left(x^2\right)\cos\left(x^2\right)$

3. Solve for the function $f(x)$ described:

   a. $f'(x) = 8x^3 + 12x + 3$ and $f(1) = 6$.

   b. $f''(x) = 20x^3 + 12x^2 + 4$, $f(0) = 8$ and $f(1) = 5$

4. (1985AB2BC1) A particle moves along the x-axis with acceleration given by $a(t) = \cos(t)$ for $t \geq 0$. At $t = 0$ the velocity $v(t)$ of the particle is 2 and the position $x(t)$ is 5.

   a. Write an expression for the velocity $v(t)$ of the particle.

   b. Write an expression for the position $x(t)$.

   c. For what values of t is the particle moving to the right? Justify your answer.

   d. Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

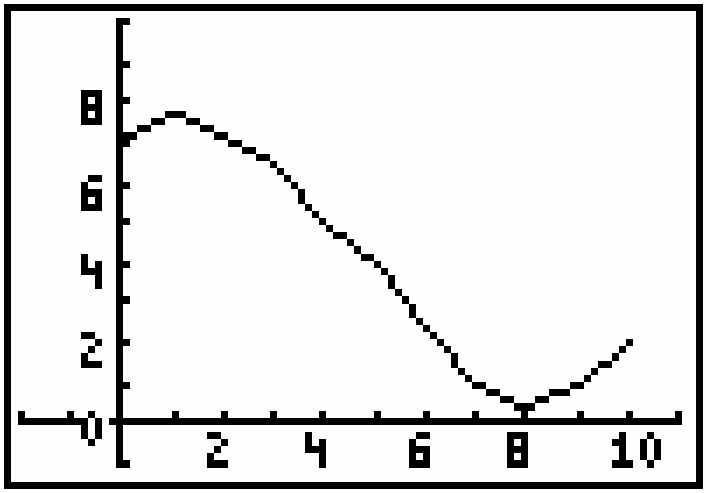
5. Given $f(x) = (x-2)^2(x+1)$. If $F(x)$ is an antiderivative of $f(x)$, find the following:

   a. the interval(s) on which $F(x)$ is increasing.

   b. the x-coordinate of any minimum or maximum point of $F(x)$.

   c. the x-coordinate of any inflection point of $F(x)$.

6. Let f be the function shown below, Let F be an antiderivative of f, and suppose that $F(0) = 10$. (The function f has a local maximum at $x = 1.14$, a point of inflection at $x = 4.67$, and a local minimum at $x = 8.19$.)

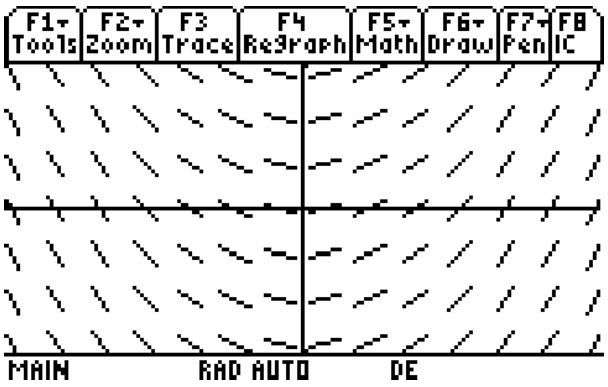


a) What is the slope of the graph of F at $x = 6$?

b) Is F concave down at $x = 6$?

c) On what interval(s) is $F(x)$ concave up?

7. The figure below shows the slope field *(aka direction field)* for the differential equation $y' = 2x$.



$-2 \le x \le 2, -2 \le y \le 2$

Sketch the solution to the differential equation that passes through the point (0,-2).

8. (1992AB2) A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t - 1)(t - 3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

   a) Find the minimum acceleration of the particle.

   b) Find the total distance traveled by the particle.

   c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

9. (1979AB3BC3) Find the maximum volume of an open box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.