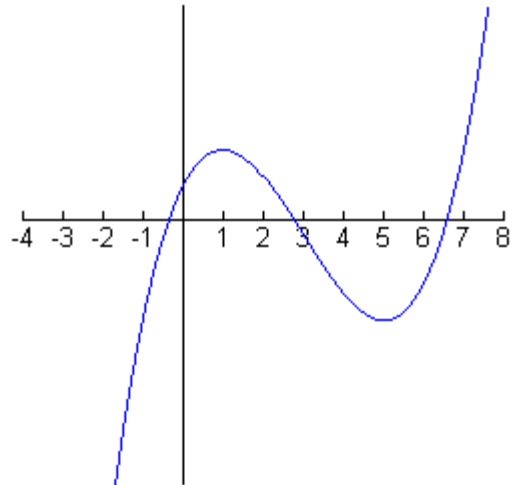


AP Calculus BC

Lesson 4.3 How derivatives affect the shape of the graph

1. Given the function $f(x) = x^3 - 9x^2 + 15x + 6$

a) Draw the graph of f on your calculator.
Adjust the window to get a "good" picture



b) On what intervals is the derivative of f positive? negative?

c) Use the tangent feature on your calculator to draw tangent lines to the graph at the points where $x = -2, -1, 0, 1,$ and 2

d) On $[-2, 2]$, is the derivative of f increasing or decreasing or neither? What evidence do you have? What is the shape of the graph of f on the interval $[-2, 2]$?

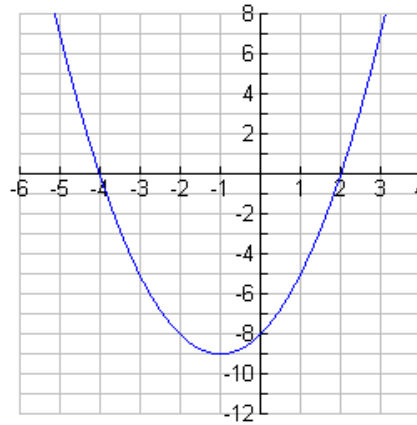
e) On $[4, 7]$, is the derivative of f increasing or decreasing or neither? What evidence do you have? What is the shape of the graph of f on the interval $[4, 7]$?

f) Draw the tangent line at $x = 3$. What is the behavior of the graph of f at $x = 3$.

g) On what intervals is the derivative of f increasing? Decreasing?

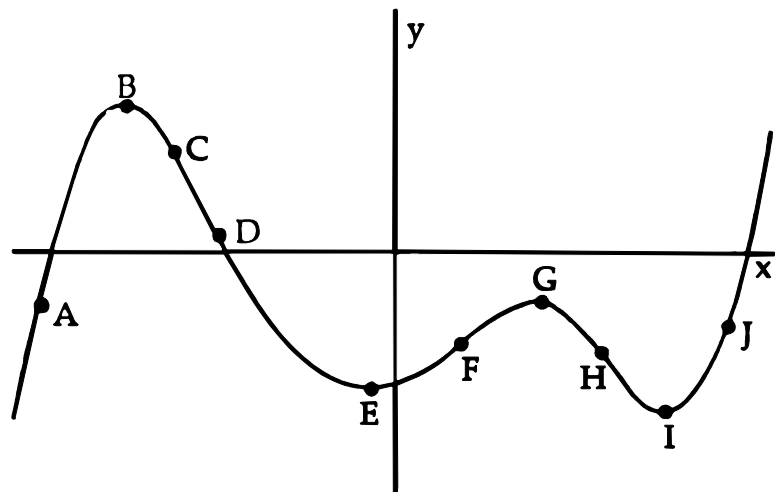
h) Using the information in b) and g), draw a sketch of the derivative of f . Then calculate the derivative of f and compare it to the sketch you made.

2. Given $f'(x) = x^2 + 2x - 8$



- Given $f(2) = 0$, write an equation for the line tangent to the graph of f at the point $(2, 0)$.
- Which is larger: $f(-4)$ or $f(2)$? Why? Find upper and lower bounds on the quantity $f(2) - f(-4)$.
- What is the behavior of the graph at $x = -4$? At $x = -2$? How do you know?
- On what intervals is f increasing? Decreasing?
- What happens on the graph of f at $x = -1$?

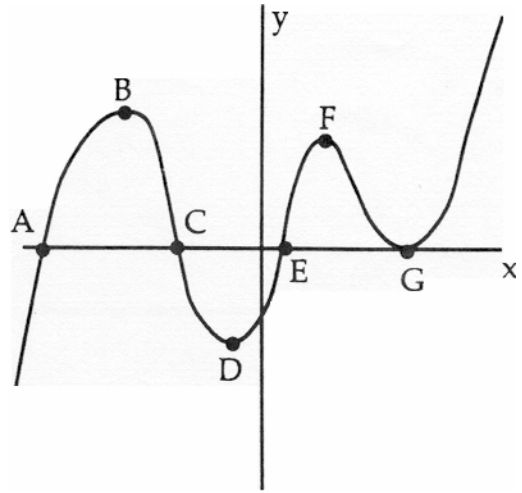
3. Given the graph of a function $f(x)$



- Which of the labeled points are stationary points?
Local maximums?
Local minimums?
Inflection points?
- On what interval(s) is f increasing?
Decreasing?
Concave up?
Concave down?
- For each of the labeled points, indicate whether $f'(x)$ is positive, negative, or zero.
- On what interval(s) is $f'(x)$ positive? Negative?
- On what interval(s) is $f'(x)$ increasing? Decreasing?
- On what interval(s) is $f''(x)$ positive? Negative?

4. Given the graph of $f'(x)$ at right:

Use lowercase letters to indicate the x -coordinates of the labeled points.



a) Explain what is happening on the graph of f at each of the labeled points.

b) On what interval(s) is f increasing?
decreasing?
Why?

c) On what intervals is f concave up? Concave down? Why?

d) On what interval(s) is $f''(x)$ positive? Negative?

5. Suppose that you have the following information about a point $(a, f(a))$ on the graph of a continuous function $f(x)$. What can you conclude?

(a) $f'(a) = 0$

(b) $f'(a) = 0$, and $f'(x) < 0$ on (b, a) and $f'(x) > 0$ on (a, c) , where $b < a < c$

(c) $f'(a) = 0$ and $f''(a) > 0$

(d) $f'(a) = 0$ and $f''(a) < 0$

(e) $f''(a) = 0$

(f) $f''(a) = 0$, and $f''(x) < 0$ on (b, a) and $f''(x) > 0$ on (a, c) , where $b < a < c$

6. Given the following information about $f(x)$, which is continuous on $[-3,3]$.

x	$(-3,-2)$	-2	$(-2,-1)$	-1	$(-1,1)$	1	$(1,2)$	2	$(2,3)$
f	+	4	+	3	+	0	-	-2	-
f'	+	DNE	-	0	-	-	-	0	+
f''	+	DNE	+	0	-	0	+	+	+

(a) Find the x -coordinate(s) of each maximum or minimum of $f(x)$. Justify your answer.

(b) Find the x -coordinate(s) of any inflection points of $f(x)$. Justify your answer.

(c) On what interval(s) is the graph of $f(x)$ increasing and concave up?

(d) Sketch a graph of f .