

AP Calculus BC

Lesson 4.4 Limits at Infinity, horizontal asymptotes

4.4(1)

Find the following limits.

1. $\lim_{x \rightarrow \infty} \frac{1}{x}$

2. $\lim_{x \rightarrow \infty} \frac{2x+1}{5x-2}$

3. $\lim_{x \rightarrow -\infty} \frac{7x^2 - 5x + 4}{3x^2 + 4x - 12}$

4. $\lim_{x \rightarrow -\infty} \frac{\sin x}{x}$

5. $\lim_{x \rightarrow -\infty} \left(3x + \frac{1}{x^3} \right)$

6. $\lim_{x \rightarrow \infty} \cos\left(\frac{1}{x}\right)$

7. $\lim_{w \rightarrow \infty} \frac{\sqrt{w^2 + 4}}{w + 4}$

8. $\lim_{x \rightarrow \infty} \left(3 - \frac{5}{\sqrt[3]{x}} \right) \left(\tan \frac{1}{x} \right)$

4.4(2)

For each of the following equations, find all vertical and horizontal asymptotes. Support your answers graphically.

1. $f(x) = \frac{2x+3}{x+7}$

2. $g(x) = \frac{4x^2}{x^2-4}$

3. $3xy - 2x - 4y - 3 = 0$

4. $x^2y + 4xy - x^2 + x + 4y - 6 = 0$

4.4(3)

Consider the function defined by $f(x) = \frac{x^2 - 5x + 3}{x - 2}$.

1. Draw a complete graph of $y = f(x)$. Describe the end behavior of the function.
2. Let $g(x) = x^2 - 5x + 3$ and $h(x) = x - 2$. Rewrite $f(x)$ as $q(x) + \frac{r(x)}{h(x)}$, where $\deg r < \deg h$.
3. Graph $y = f(x)$ and $y = q(x)$ on the window $[-6, 12]$ by $[-15, 15]$. Predict what happens if you zoom out.
4. Find an equation of the function which describes the end behavior of $f(x)$.

4.4(4)

Consider the function defined by $f(x) = \frac{g(x)}{h(x)}$, where $g(x) = x^3 + 8x^2 + 7x - 16$

and $h(x) = x + 3$.

1. Draw a complete graph of $y = f(x)$. Describe the end behavior of the function.
2. Rewrite $f(x)$ as $f(x) = q(x) + \frac{r(x)}{h(x)}$, where $\deg r < \deg h$.
3. Graph $y = f(x)$ and $y = q(x)$ on the window $[-16,10]$ by $[-25,25]$. Predict what happens if you zoom out.
4. Find an equation which describes the end behavior of $f(x)$.

4.4(5)

Find the end behavior asymptote and all vertical asymptotes for each of the following functions.

1. $f(x) = \frac{5x-6}{3x+9}$

2. $g(x) = \frac{2x^3-5x^2-3x+5}{x-4}$

3. $h(x) = \frac{2x^3-5x^2-3x+5}{x^2-2x-8}$

4.4(6)

Evaluate each limit.

1. $\lim_{x \rightarrow \infty} (\sqrt{x^2+1} - x)$

2. $\lim_{x \rightarrow \infty} (\sqrt{x^2+x} - x)$