AP Calculus BC
Lesson 4.9 Newton's Method

4.9(1) Consider the function $f(x) = -x^3 + x + 3$

The following table gives some values of *f* and *f'*

x	1	2.5	1.9296	1.7079	1.6726
f(x)	3	-10.125	-2.255	-.2737	-.0067
f ' (x)	-2	-17.75	-10.170	-7.7505	-7.393

a) Graph *f* using the interval x:[-2,5] , y:[-30,5] Notice that *f* has a zero somewhere between 1 and 2.

b) Using the table data write the equation of the tangent line to *f* at $x = 1$ and then graph this tangent line also on your graph. Find the *x*- intercept of this tangent line.

c) Write the equation of the tangent line to *f* at the point whose *x*-coordinate is the *x*-intercept you found in part (b). Hint: Use the table data. Graph this also on your graph. Find the *x*-intercept of this tangent line.

d) Repeat part c using the *x*-intercept you found in part c. Graph this also on your graph. Find the *x*-intercept of this third tangent line.

e) What do you notice about these successive graphs and their *x*-intercepts? What do you think would happen if you continued this process several more times?

f) Using the root finding capability of your calculator, calculate the root of *f* and compare it to the last *x*-intercept in the table. What do you notice? What is happening to successive *y*-values in the table?

4.9(2) a) Given a function y = f(x) , write in symbolic language the equation of the tangent line to f at the point $\left(x_0, f(x_0)\right)$

b) Using the equation you wrote in part (a) , find a symbolic expression for the x-intercept of this tangent line. Set this expression equal to x_1

c) Using this symbolic equation we can think of this formula as a recursive formula – oh boy math 324 the red book -. If we use the output value of this formula as the next input and call the new output x_2 , what is the formula now?

d) Generalize to a formula that would result after n-iterations. This formula is called Newton's Method and really just calculates successive x-intercepts of successive tangent lines.

4.9(3) a) Using the function from problem #1 do the following:

In $y_1 = -x^3 + x + 3$, i.e. f(x) and in $y_2 = -3x^2 + 1$, i.e. f ' (x)

Now go to the home screen and type

$1 \to$ x <Enter>

$x - y_1(x)/y_2(x) \to x$ <Enter> {This is the Newton Method formula}

You might want to get an approximate answer (diamond enter)…

Now press <Enter> 3 more times. Compare these values to the x-values in the table in problem #1. Hit <Enter> a few more times until the iteration process stabilizes.

b) What does this stable value represent? Why?

4.9(4) Use Newton's Method to find a solution to the equation $x \cdot \tan(x) = 1$, which has a solution somewhere in the interval $[0, \pi/2]$ Use $x_0 = 1$.

4.9(5) Let f(x) = $x^2 - a$

a) Explain why between the two numbers x and a/x is the value of $\sqrt{a}$.
b) Show that the Newton's Method applied to f, starting at x = x_0 gives the

iteration function $x_n = \dfrac{x_{n-1} + \dfrac{a}{x_{n-1}}}{2}$

c) Use this Newton's method with a = 2 on your calculator.
d) What happens if start with $x_0 = \sqrt{a}$?

4.9(6) The Compound Amount formula calculates the total value, V of an investment if p dollars are invested annually at an effective annual rate r for n years. The formula is:

$$V(n) = \frac{p}{r}\left((1+r)^{n+1} - 1\right)$$

a) If p = $2000, n = 30 years and you want to have a value V = $300,000 after those 30 years, what effective annual rate do you need. Substitute into this equation and simplify so you have no fractions.

b) Use Newton's method to find the root r of this equation. Guess an initial value of 6% to begin the iteration.

4.9(7) a) Consider f(x) = $x^3 - 6x^2 + 7x + 2$ and use Newton's Method on your calculator with $x_0 = 1$ to find a root of f.

b) What happens and why does it happen?
c) What other bad things might happen if you pick a "bad" initial guess for the root?