AP Calculus BC
Lesson 5.1 Sums and Areas

1. Evaluate each of the following sums without using your calculator:

(a) $\sum_{i=0}^{5} i^2$

(b) $\sum_{k=0}^{5} \sin\left(\frac{k\pi}{2}\right)$

(c) $\sum_{j=1}^{n} j$

(d) $\sum_{k=1}^{n} a$

(e) $\sum_{k=1}^{n} k^2$

2. Given the function $f(x) = x^2 + 2x$ defined on the closed interval [1,5]. Approximate the area bounded by *f*, the *x*-axis, and the vertical lines $x = 1$ and $x = 5$ by dividing the interval into 4 subintervals of equal width.

(a) Use rectangles and determine the height by using the *y*-value at the left hand endpoint of each subinterval.

(b) Use rectangles and determine the height by using the *y*-value at the right hand endpoint of each subinterval.

(c) Use rectangles and determine the height by using the *y*-value at the midpoint of each subinterval.

(d) Use trapezoids to approximate the area.

(e) Which of these approximations overestimate the area? Underestimate the area?

3. Use the function $f(x) = x^2 + 2x$ on the interval [1,5]. Divide the interval into n subintervals. Using the y-value at the right endpoint of each interval, do the following:

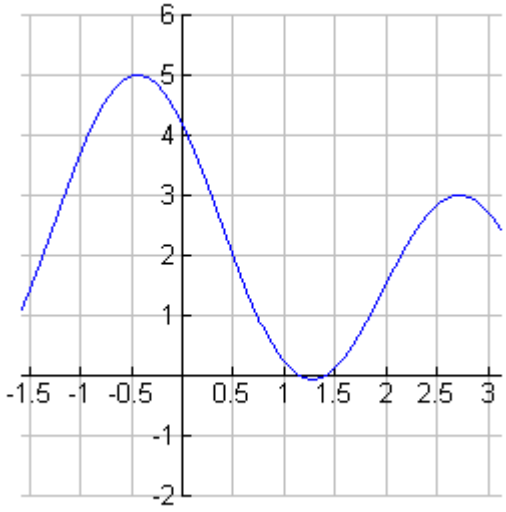
(a) Write, using sigma notation, an approximation for the area under the curve using n subintervals.

(b) Simplify the sum in part (a).

(c) Evaluate the limit of your expression in part (b) as n approaches ∞.

(d) What area does this limit represent?

4. Using the function $f(x)$ at right, estimate the area bounded by the function $f(x)$, the x-axis, and the vertical lines $x = -1$ and $x = 2$. Use six right endpoint rectangles.



5. Each of the following represents an area. Describe the area and use your calculator to evaluate the expression.

(a) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^2\left(\frac{1}{n}\right)$

(b) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{5k}{n}\right)^2\left(\frac{5}{n}\right)$

(c) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(4+\frac{k}{n}\right)^2\left(\frac{1}{n}\right)$

(d) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{2k}{n}\right)^2\left(\frac{2}{n}\right)$

(e) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{n}\right)\sin\left(\frac{k}{n}\right)$

(f) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{3}{n}\right)\sin\left(\frac{3k}{n}\right)$

(g) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{n}\right)\sin\left(2+\frac{k}{n}\right)$

(h) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{7}{n}\right)\sin\left(5+\frac{7k}{n}\right)$