AP Calculus BC

Lesson 5.3 **The Fundamental Theorem of Calculus**

1. Find the average value of the function $f(x) = \sqrt{25 - x^2}$ on the interval [–5, 5].

   It might be useful to recall that the average value of $f(x)$ on $[a,b]$ = $\dfrac{\int_a^b f(x)dx}{b-a}$.

The Fundamental Theorem of Calculus states that $\dfrac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$.

2. Use the limit definition of the derivative to prove the Fundamental Theorem of Calculus.

3. Use the fundamental theorem to find the derivative of $F(x)$.

   (a) $F(x) = \int_0^x \sqrt{t}\,dt$

   (b) $F(x) = \int_3^x \sin(t)dt$

   (c) $F(x) = \int_x^3 \sin(t)dt$

   (d) $F(x) = \int_5^{x^2} \cos(t)dt$

   (e) $F(x) = \int_{2x}^{3-x^2} e^t dt$

4. Find each indefinite integral:

(a) $\int 3x^2 dx$

(b) $\int \cos(x)\, dx$

(c) $\int \frac{3}{\mathrm{x}} dx$

(d) $\int \frac{1}{1+\mathrm{x}^2} dx$

(e) $\int \mathrm{e}^{2x} dx$

5. Find the value of the definite integral using two methods:

(1) $\int_a^b f(t)\ dt\ = F(b) - F(a)$ where F is any antiderivative of f,

and

(2) your calculator.

(a) $\int_0^2 (4 - x^2) dx$

(b) $\int_0^\pi \sin(x) dx$

6. Find $\frac{dy}{dx}$ for each function given:

(a) $y = \int_3^x e^t \, dt$

(b) $y = \int_0^{x^2} t\sin(t) \, dt$

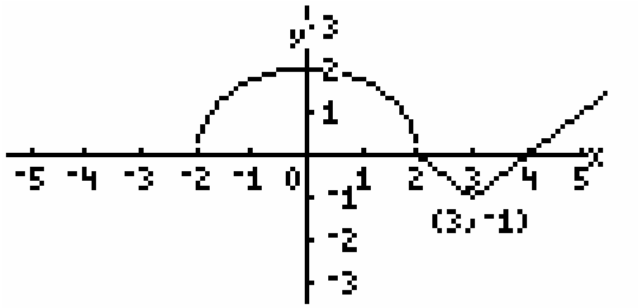
(c) $y = \int_{-4x}^3 \ln(t) \, dt$

(d) $y = \int_{2x}^{4x^2} \frac{1}{t^2+1} \, dt$

7. Solve for a if $\int_0^a (6x - x^2)dx = 4$.

8. Find the area inside the parabola $x = (y-2)^2$ from its vertex to the line $x = 4$.

9. (1997AB5BC5)



The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)dt$.

(a) Find $g(3)$.

(b) Find all values of x on the open interval (-2,5) at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval (-2,5). Justify your answer.