AP Calculus BC Lesson 5.4 More with Integrals

1. Find the area of the region bounded (on the left) by the y-axis and (on the right) by the parabola $x = 3y - y^2$.

- 2. [1996AB3BC3] The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.
 - a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.
 - b) Find the average rate of consumption over the 10-year period beginning January 1, 1983. Indicate units of measure.
 - c) Use trapezoids with four equal subdivisions to estimate $\int_{5}^{7} S(t) dt$.
 - d) Using correct units, explain the meaning of $\int_{5}^{7} S(t) dt$ in terms of cola consumption.

- 3. Evaluate each of the following without using your calculator.
 - (a) $\int (x^3 4x^2 + 5) dx$ (b) $\int 5 \sec(x) \tan(x) dx$

(c)
$$\int \cos(2x) dx$$
 (d) $\int \frac{3}{x^4} dx$

(e)
$$\int 3x(x^2-4)dx$$
 (f) $\int \frac{x^4-3x^2+2x\sin(x)}{x}dx$

- 4. Given $\int \sec^2(x) \tan(x) dx$.
 - (a) Show that a general solution of this problem is $f(x) = \frac{1}{2} \tan^2(x) + C_1$.

(b) Show that a general solution of this problem is $g(x) = \frac{1}{2}\sec^2(x) + C_2$.

- (c) How can there be two different solutions for this one problem? Explain.
- 5. Use the identity $\cos(2x) = \cos^2(x) \sin^2(x) = 2\cos^2(x) 1 = 1 2\sin^2(x)$ to find each of the following antiderivatives.
 - (a) $\int \cos^2(x) dx$
 - (b) $\int \sin^2(x) dx$