AP Calculus BC
Chapter 6
Assignment 61

1. The air density (in kg/m^3) h meters above the surface of Earth is $P = f(h)$. Find the mass of a cylindrical column of air 2 meters in diameter and 25 kilometers high.

2. Find the mass of the column of air in Problem 1 if the density of air at height h meters is given by $P = f(h) = 1.28e^{-0.000124h}$ kg/m^3.

3. A rod has length 2 meters. At a distance x meters from its left end, the density of the rod is given by $\rho(x) = 2 + 6x$ g/m.

   a. Write a Riemann sum approximating the total mass of the rod.
   b. Find the exact mass by converting the sum into an integral.
   c. The *center of mass* of a rod is the point at which you would place a pivot so that the rod balances. To find the position of the center of mass of a rod, you first calculate its *moment*, and then divide the moment by the total mass. If the rod lies along the x-axis between a and b, the moment is $\int_a^b x\rho(x)dx$, where $\rho(x)$ is its density at a position x. Find the moment of the rod.

4. Circle City, a typical metropolis, is very densely populated near its center, and its population gradually thins out toward the city limits. In fact, its population density is $10000(3 - r)$ people/square mile at distance r miles from the center.

   a. Assuming that the population density at the city limits is zero, find the radius of the city.
   b. What is the population of the city?

5. Water is flowing in a cylindrical pipe of radius 1 inch. Because water is viscous and sticks to the pipe, the rate of flow varies with the distance from the center. The speed of the water at a distance r inches from the center is $10(1 - r^2)$ inches per second. What is the rate (in cubic inches per second) at which water is flowing through the pipe?