AP Calculus BC
Lessons 6.2 Volume using slices, disks and washers

1. A three dimensional object has a square base, defined by $1 \le x \le 7$ and $0 \le y \le 6$. The height of the object at any point (x,y) in the base is a function of x alone and is given by $h(x) = x^2 + 1$.

   (a) Approximate the volume of the object by dividing the base into three vertical strips, each of width 2, and using the constant height obtained by using the smallest value of x in each strip.

   (b) Suppose the base is divided into n vertical strips, each of width $\Delta x \left(\Delta x = \frac{7-1}{n} \right)$, and we use the constant height obtained by using the smallest value of x in each strip. Write a Riemann sum that approximates the volume of the object.

   (c) Write an integral for the volume of the object.

   (d) Find the volume of the object.

2. Consider the function defined by $y = 2x$ for $0 \le x \le 3$.

(a) Rotate this curve around the *x*-axis. Describe the solid that is generated.

(b) Slice the solid generated perpendicular to the *x*-axis. What is the shape of each cross section? What is the area of the base of each cross section?

(c) Suppose that each cross section has width $\Delta x = \frac{b-a}{n}$. What is the approximate volume of each cross section?

(d) Using the *n* cross sections of this solid, find the approximate volume of the solid.

(e) If *A(x)* is the area of the base of each cross section and Δx is the width of each cross section, give a geometric interpretation of $\sum_{k=1}^{n} A(x)\Delta x$.

(f) Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} A(x)\Delta x$.

(g) Give a geometric interpretation of $\int_0^3 A(x)dx$.

3. Find the volume of the solid generated by revolving the curve defined by $f(x) = \sqrt{r^2 - x^2}$ around the *x*-axis.

4. Find the volume of the solid obtained by rotating around the *x*-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

5. Find the volume of the solid obtained by revolving around the *y*-axis the region bounded by $y = x^3$, $y = 8$, and $x = 0$.

6. The region R is bounded by the curves $y = x^2$ and $y = x$.

   (a) Find the volume of the solid generated if R is revolved around the x-axis.

   (b) Find the volume of the solid generated if R is revolved around the y-axis.

   (c) Find the volume of the solid generated if R is revolved around the line $y = 2$.

7. A square pyramid has base length L and height h. The vertex of the pyramid is at the origin and the central axis is along the x-axis. Find the volume of the pyramid.

8. A solid has a circular base of radius 1. Parallel cross-sections perpendicular to the base are equilateral triangles. Find the volume of the solid.