AP Calculus BC
Lessons 6.3 Volume using cylindrical shells

1. The region in the first quadrant bounded by the parabola $y = x^2$, the *y*-axis, and the line $y = 4$ is revolved around the *x*-axis.

   (a) Find the volume of the solid generated by using washers.

   (b) Find the volume of the solid generated by using cylindrical shells.

2. A solid is obtained by rotating around the *y*-axis the region between $y = x$ and the parabola $y = x^2$. Find the volume using cylindrical shells.

3. The region bounded by $y = x$, $y = 0$, $x = 2$, and $x = 4$ is revolved about the line $x = 1$. Find the volume of the solid generated.

4. Each integral represents the volume of a solid. Describe the solid.

   1. $\pi\int_0^{\pi/4} \tan^2 x\, dx$

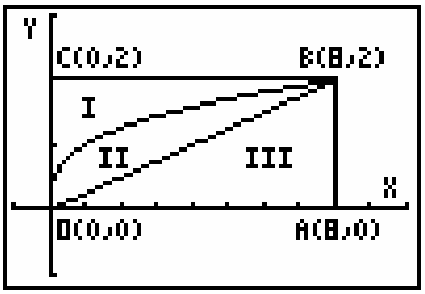
   2. $\pi\int_0^1 \left(y - y^2\right) dy$

   3. $\pi\int_2^4 [16 - (x-2)^2]dx$

5. The region in the first quadrant bounded by the parabola $y = x^2$, the y-axis, and the line $y = 1$ is revolved about the line $x = 2$ to generate a solid. Find the volume of the solid.

6. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = x(x-1)^2$ and $y = 0$.

7. The curve separating regions I and II is the graph of $y = \sqrt[3]{x}$. For each of the following find the volume generated by revolving the given region about the given line.



(a) III about $\overline{OA}$

(b) III about $\overline{AB}$

(c) I about $\overline{AB}$

(d) III about $\overline{BC}$

(e) II about $\overline{OA}$

(f) II about $\overline{OC}$

(g) II about $\overline{BC}$

(h) II about $\overline{AB}$

(i) I about $\overline{OA}$

(j) I about $\overline{OC}$

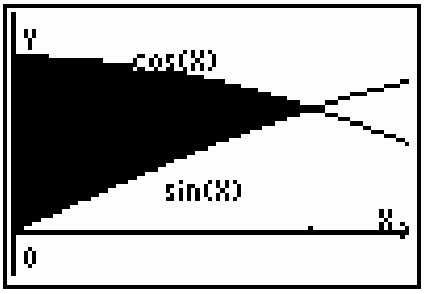
(k) I about $\overline{BC}$

(l) I about $\overline{AB}$

(m) II about $y = 4$

(n) II about $x = 4$

8. (1991BC3) Let R be the shaded in the first quadrant enclosed by the y-axis and the graphs of $y = \sin x$ and $y = \cos x$, as shown in the figure below.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

9. (1981AB2) Let *R* be the region in the **first quadrant** enclosed by the graphs of $y = 4x^2$, $y = 3x$, and the *y*-axis.

   (a) Find the area of region *R*.

   (b) Find the volume of the solid formed by revolving the region *R* about the *x*-axis.

10. (1989AB2) Let *R* be the region in the first quadrant enclosed by the graph of $y = \sqrt{6x+4}$, the line $y = 2x$, and the *y*-axis.

   (a) Find the area of *R*.

   (b) Set up, but **DO NOT INTEGRATE**, an integral expression in terms of a single variable for the volume of the solid generated when *R* is revolved about the ***x*-axis.**

   (c) Set up, but **DO NOT INTEGRATE**, an integral expression in terms of a single variable for the volume of the solid generated when *R* is revolved about the ***y*-axis.**