AP Calculus BC
Logarithm and Exponent Practice Problems

1. For each function, find $\frac{dy}{dx}$. (No calculator!)

a) $y = \sec^2\left(e^{4\ln x}\right)$

b) $y = \ln\left(\tan(3x)\right)$

c) $y = 2 \cdot 3^{x-1}$

d) $y = (\ln 5)5^x$

e) $y = 3\log_2 x$

2. Find a general antiderivative for each function. (No Calculator!)

a) $e^{\tan(x)}\sec^2(x)$

b) $y = e^{2x}\cos\left(e^{2x}\right)$

c) $y = (\ln 5)^x$

3. Evaluate each limit without a calculator:

a) $\lim\limits_{x \to 0} \dfrac{2^{3+x} - 2^3}{x}$

b) $\lim\limits_{x \to 0} \dfrac{\log_5(25+x) - 2}{x}$

4. (1999AB4) Suppose that the function *f* has a continuous second derivative for all *x*, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let *g* be a function whose derivative is given by $g'(x) = e^{-2x}\left(3f(x) + 2f'(x)\right)$ for all *x*.

a) Write an equation of the line tangent to the graph of *f* at the point where *x* = 0.

b) Is there sufficient information to determine whether or not the graph of *f* has a point of inflection when *x* = 0? Explain your answer.

c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of *g* at the point where *x* = 0.

d) Show that $g''(x) = e^{-2x}\left(-6f(x) - f'(x) + 2f''(x)\right)$. Does *g* have a local maximum at *x* = 0? Justify your answer.

5. (1988BC1) Let f be the function defined by $f(x) = \left(x^2 - 3\right)e^x$ for all real numbers x.

(a) For what values of x is f increasing?

(b) Find the x-coordinate of each point of inflection of f.

(c) Find the x- and y-coordinates of the point, if any, where $f(x)$ attains its absolute minimum.

6. Find the area of the largest rectangle that has one side on the positive x-axis, one side on the negative y-axis, a vertex at the origin, and a vertex on the curve $y = \ln x$.

7. Does there exist a real number a such that the line tangent to the curve $y = e^x$ at $x = a$ passes through the origin? If so, find it. If not, explain why there is no such number.