AP Calculus BC
Lessons 7.1 and 7.2

1. Draw the graph of $y = .125x^3$ and the graph of its inverse function using a window of $-6 \le x \le 6$ and $-4 \le y \le 4$.

2. Pick the point (2,1) on the graph of $y = .125x^3$. Write an equation of the tangent line to $y = .125x^3$ at this point and draw this tangent line on your calculator.

3. Write an equation for the tangent line to the inverse function at the point (1,2). Draw this tangent line with your calculator.

4. What do you notice about the two tangent lines that you drew? Generalize.

5. If $f(x) = x^2 - 5x - 2$, $x > 2.5$, find the derivative of $f^{-1}(4)$.

6. If $f(x) = \sqrt{x}$, find $\dfrac{d\left(f^{-1}(x)\right)}{dx}$.

7. What is $\lim_{h \to 0} \frac{e^h - 1}{h}$? Explain your reasoning.

8. Write the limit definition of the derivative for the function $f(x) = a^x$. Simplify as much as possible.

9. Find $f'(x)$ if $f(x) = e^x$.

10. Find $\lim_{h \to 0} \frac{2^h - 1}{h}$ graphically.

11. Find $\lim_{h \to 0} \frac{3^h - 1}{h}$ graphically.

12. What is the slope of a^x when $x = 0$?

13. Can you predict $\frac{d}{dx}\left(a^x\right)$?

14. Show that $a^x = e^{x\ln(a)}$.

15. Use the result in problem 1 to find $\frac{d\left(a^x\right)}{dx}$.

16. Find $\frac{dy}{dx}$ for each of the following:

a. $y = e^{2x}$

b. $y = 3^x + x^3$

c. $y = 5 \cdot 3^{x-1}$

17. Calculate $\ln(e^x)$ and $e^{\ln(x)}$.

18. Draw the graphs of $y = \ln(e^x)$ and $y = e^{\ln(x)}$. Are they the same? Explain.

19. Calculate the value of each of the following without a calculator

a. $e^{4\ln(5)}$ b. $\ln(\sqrt[4]{e})$ c. k, if $e^{k^2-4} = 1$

d. $e^{x + \ln(3)}$ e. $e^{2x - 2\ln(3)}$ f. $e^{-4\ln(x)}$

20. Given the following functions:

a. $f(x) = e^x + x^e + e$ b. $g(x) = 2\cdot 3^{x-1}$

c. $h(x) = x^{\ln(5)}$

Find the first derivative of each function.

Find the second derivative of each function.

Find an antiderivative of each function.

21. One model for the way diseases spread assumes that the rate at which the number y of infected people changes is proportional to y itself. The more infected people there are, the faster the disease will spread. The fewer there are, the slower it will spread. If y_0 is the number of infected people at time $t = 0$, then the number of infected people at any time in the near future will be about $y = y_0e^{kt}$. Suppose that in the course of any given year the number of cases worldwide is reduced by 20%. If there are 10000 known cases today, how long will it be before the number of cases is reduced to 1000?