BC Calculus
Doc

Name___________________________

Rollem'

(a.k.a. "Fun with logs!")

Calculator Free Questions:

1. Solve for x: $\ln(\ln x) = 1$

2. Solve for x: $2\ln x = \ln 2 + \ln(3x-4)$

3. If $\log_2 x = \lambda$, then what is $\log_{\frac{1}{2}} x$?

4. If $\log_b x = \lambda$, then what is $\log_{\frac{1}{b}} x$?

5. If $\log_b x = \lambda$, then what is $\log_{b^2} x$?

Calculator encumbered questions:

6. $\log_2 3$ to eight decimal places:

7. Suppose the log buttons on your calculator were broken but you knew the answer to number 6 above:

   a) Determine $\log_2\left(\frac{3}{2}\right)$ to 4 decimal places.

   b) Determine $\log_2 36$ to 4 decimal places.

   c) Determine $\log_3 2$ to 4 decimal places.

8. The geologist C.F. Richter defined the magnitude of an earthquake to be $\log_{10}\left(\frac{I}{S}\right)$, where *I* is the intensity of the quake (measured by the amplitude of a seismograph 100 km from the epicenter) and *S* is the intensity of a "standard" quake (where the amplitude is only 1 micron = 10^{-4} cm). The 1989 Loma Prieta earthquake that shook San Francisco had a magnitude of 7.1 on the Richter scale. The 1906 San Francisco earthquake was 16 times more intense. What was its magnitude on the Richter scale?

9. A sound so faint that it can just be heard has intensity $I_0 = 10^{-12}$watt/m^2 at a frequency of 1000 hertz (Hz). The loudness, in decibels (dB), of the sound with intensity I is then defined to be $L = 10\log_{10}\left(\frac{I}{I_0}\right)$. Amplified rock music is measure at 120 dB, whereas the noise from a motor-driven lawn mower is measured at 106 dB. Find the ratio of the intensity of rock music to that of the mower. (In other words, how many times louder is it?)

10. Suppose that you invest $1 at 100% interest for one year. Find the amount that your investment is worth if you compound the interest:

a) Once a year

b) Twice a year

c) Monthly

d) Weekly

e) Daily

f) Hourly

g) Every minute

h) Every second

i) Every millionth of a second (yes do it, the answer may surprise you)

j) The limit as the number of periods goes to ∞.

11. We realize that $\log_2 3$ has a lot of digits. However, the calculator can only show a finite number of them. It is possible that it eventually terminates or repeats. In other words, the calculator cannot help us determine whether it is rational or not. Let us assume that it **is** rational and see what happens:

a) Assume $\log_2 3 = \frac{a}{b}$. Show that a and b must then satisfy $2^a = 3^b$

b) Notice that a = 0, b = 0 satisfies $2^a = 3^b$. Show that this fact doesn't hold in this case.

c) Find $a \neq 0$ and $b \neq 0$ that satisfy $2^a = 3^b$, or show that no such solution exists.

d) Is $\log_2 3$ rational or irrational? Explain.