AP Calculus BC
Lesson 7.4 Derivatives of log functions

1. Consider the function $f(x) = e^x$ and the relationship between the slopes of the function and its inverse.

   a. Find the value of $\left(f^{-1}(x)\right)'$ at the point where x = 2

   b. Find the value of $\left(f^{-1}(x)\right)'$ at the point where x = 5

   c. Find a function rule for $\left(f^{-1}(x)\right)'$.

2. a. Calculate $\ln(e^x)$ and $e^{\ln(x)}$.

   b. Draw the graphs of $y = \ln(e^x)$ and $y = e^{\ln(x)}$. Are they the same? Explain.

3. Define $y = \log_a(x)$ to be the inverse function for $y = a^x$, that is, $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$.

   a. Draw the graph of $y = \log_7(x)$ using your graphing calculator.

   b. Find $\frac{d}{dx}(\log_7(x))$.

4. Find $\frac{dy}{dx}$ for each of the following.

a. $y = 2\ln(x)$

b. $y = 2\log_2(x)$

c. $y = -5\log_{10}(3x) + 7e^{2x}$

d. $y = \frac{\log_4(x)}{\log_4(5)}$

5. Let $f(x) = \ln(x) - e^{x-1}$.

a. Show that f achieves its global maximum value when $x = 1$.

b. Show that f is concave down everywhere on its domain.

6. Find the most general antiderivative of each function:

a. $f(x) = \frac{3}{x} + e^{2x} - 2x \cdot 3^{x^2} + 4$

b. $g(x) = \sin x \cdot e^{\cos x} - 5^{4\log_5 x} + 2e$