AP Calculus BC
Lesson 7.5 Inverse Trig Functions

1. Draw the graph of each of the following functions. Give the domain of each function and the range of each function.

a. $y = \sin^{-1}(x)$

b. $y = \cot^{-1}(x) = \dfrac{\pi}{2} - \tan^{-1}(x)$

c. $y = \cos^{-1}(x)$

d. $y = \sec^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right)$

e. $y = \tan^{-1}(x)$

f. $y = \csc^{-1}(x) = \sin^{-1}\left(\dfrac{1}{x}\right)$

2. Determine the value of each of the following without a calculator

a. $\cos^{-1}\left(\frac{1}{2}\right)$

b. $\cot^{-1}(-1)$

c. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

d. $\sin(\alpha)$, $\cos(\alpha)$, and $\tan(\alpha)$ if $\alpha = \tan^{-1}(-\sqrt{3})$

e. $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

f. $\sin\left(\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$

g. $\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right)$

h. $\sin\left(2\tan^{-1}\left(\frac{x}{3}\right)\right)$

3. a. If $y = \sin^{-1}(x)$, then an equivalent statement is $\sin(y) = x$. Why?

b. Use implicit differentiation on $\sin(y) = x$, to find a formula for dy/dx, where $y = \sin^{-1}(x)$.
Note that your final answer must be a function of x.
Hint: you may need to use a trigonometric identity.

c. Try the same technique as in numbers 1 and 2, but begin with $y = \tan^{-1}(x)$.

d. Try the same technique as in numbers 1 and 2, but begin with $y = \sec^{-1}(x)$.

4. Evaluate each of the following:

a. $\frac{d}{dx}\left(\sin^{-1}(\ln(x))\right)$

b. $\frac{d}{dx}\left(\tan^{-1}(x^3)\right)$

c. $\frac{d}{dx}\left(\sec^{-1}(\sin(x))\right)$

d. $\frac{d}{dx}\left(\cos^{-1}(\sqrt{x+5})\right)$

5. In problem #3 you found formulas for the derivatives of $y = \sin^{-1}(x)$, $y = \tan^{-1}(x)$, and $y = \sec^{-1}(x)$. Use these formulas to find an antiderivative for each of the following:

a. $\dfrac{1}{1+x^2}$

b. $\dfrac{1}{x\sqrt{x^2-1}}$

c. $\dfrac{1}{\sqrt{1-x^2}}$

d. $\dfrac{1}{\sqrt{16-x^2}}$

e. $\dfrac{2x}{1+x^4}$

f. $\dfrac{e^x}{1+e^{2x}}$

g. $\dfrac{1}{x\sqrt{16x^2-1}}$

h. $\dfrac{4x}{\sqrt{4-x^4}}$