BC Calculus Name____________________
DrC

Riemann "Summary"

Left Sums

$$\int_0^a f(x)dx = \lim_{n\to\infty}\left(\frac{a}{n}\bullet\sum_{k=0}^{n-1} f\left(\frac{a}{n}k\right)\right) \textbf{ and } \int_a^b f(x)dx = \int_0^{b-a} f(x+a)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\bullet\sum_{k=0}^{n-1} f\left(a+\frac{b-a}{n}k\right)\right)$$

Right Sums

$$\int_0^a f(x)dx = \lim_{n\to\infty}\left(\frac{a}{n}\bullet\sum_{k=1}^{n} f\left(\frac{a}{n}k\right)\right) \textbf{ and } \int_a^b f(x)dx = \int_0^{b-a} f(x+a)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\bullet\sum_{k=1}^{n} f\left(a+\frac{b-a}{n}k\right)\right)$$

Midpoint Sums

$$\int_0^a f(x)dx = \lim_{n\to\infty}\left(\frac{a}{n}\bullet\sum_{k=0}^{n-1} f\left(\frac{a}{2n}+\frac{a}{n}k\right)\right) \textbf{ or } \int_0^a f(x)dx = \lim_{n\to\infty}\left(\frac{a}{n}\bullet\sum_{k=1}^{n} f\left(\frac{a}{n}k-\frac{a}{2n}\right)\right) \textbf{ and}$$

$$\int_a^b f(x)dx = \int_0^{b-a} f(x+a)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\bullet\sum_{k=0}^{n-1} f\left(a+\frac{b-a}{2n}+\frac{b-a}{n}k\right)\right) \textbf{ or}$$

$$\int_a^b f(x)dx = \int_0^{b-a} f(x+a)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\bullet\sum_{k=1}^{n} f\left(a-\frac{b-a}{2n}+\frac{b-a}{n}k\right)\right)$$

Trapezoid Sums (Equivalent to the average of Left and Right Sums)

$$\int_0^a f(x)dx = \lim_{n\to\infty}\left(\frac{a}{n}\bullet\sum_{k=0}^{n-1}\frac{f\left(\frac{a}{n}k\right)+f\left(\frac{a}{n}(k+1)\right)}{2}\right) \textbf{ and}$$

$$\int_a^b f(x)dx = \int_0^{b-a} f(x+a)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\bullet\sum_{k=0}^{n-1}\frac{f\left(a+\frac{b-a}{n}k\right)+f\left(a+\frac{b-a}{n}(k+1)\right)}{2}\right)$$