AP Calculus BC
Lesson 8.1 - Integration by Parts

Notes: From the product rule for derivatives, if u and v are differentiable functions of x then

$\dfrac{d(uv)}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$. Considering these as differentials and multiplying through by dx and
rearranging yields $udv = d(uv) - vdu$ and finally integrating both sides gives us the integration by
parts formula $\int udv = uv - \int vdu$, which can be used to evaluate more complicated integrals. ☺

8.1(1) Use two methods to antidifferentiate the following:

a) $\int x\cos(x)dx$

b) $\int xe^x dx$

c) $\int x^2 e^x dx$

d) $\int x\sec^2(x)dx$

e) $\int \tan^{-1}(x)dx$

f) $\int x^3 e^{x^2} dx$

8.1(2) Use integration by parts to evaluate the integral obtained for the following problem.

Find the volume of the solid formed by revolving about the y-axis the region in the first quadrant bounded by the coordinate axes, the curve $y = e^{-x}$ and the line $x = 1$.

8.1(3) Show by using integration by parts that $\displaystyle\int \tan^n (x)dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)dx$

8.1(4) If $\displaystyle\int f(x)\sin(x)dx = -f(x)\cos(x) + \int 3x^2 \cos(x)dx$, then $f(x)$ could be

a) $3x^2$ b) x^3 c) $-x^3$ d) $\sin(x)$ e) $\cos(x)$

8.1(5) Use two methods to evaluate each of the following:

a) $\displaystyle\int e^x \cos(x)dx$

b) $\displaystyle\int \cos(\sqrt{x})dx$

c) $\displaystyle\int x^2 \sin(3x)dx$

d) $\int e^{3x} \sin(4x)dx$

e) $\int e^{-x} \cos(x)dx$

8.1(6) Show by using integration by parts that $\int (\ln(x))^n dx = x(\ln(x))^n - n\int (\ln(x))^{n-1} dx$.

8.1(7) Find the volume of the solid described: Its base is bounded by the graphs of $y = \sin x$ and $y = 0$ from $0 \le x \le \pi$, and cross sections perpendicular to the x-axis are rectangles with height given by $y = e^x$.

8.1(8) More Practice Problems!! Evaluate the integral without using a calculator:

a) $\int \sin(\ln x)\, dx$

b) $\int \ln x\, dx$

c) $\int \log_2 x\, dx$

d) $\int x^3 \cos x\, dx$

e) $\int x \ln x\, dx$

f) $\int e^{3x} \cos(2x)\, dx$

g) $\int x^n \ln x\, dx$

h) $\int \sin^{-1} x\, dx$

i) $\int \sin^6 x\, dx$

j) $\int \frac{\ln x}{x^5}\, dx$

k) $\int x \sec^2 x\, dx$

l) $\int \csc^{-1} x\, dx$