**AP Calculus BC**
**Lesson 8.2 Trigonometric Antiderivatives**

8.2(1) Integrating odd powers of sin($x$), cos($x$), or combinations containing at least one odd power of either sin(x) or cos(x):

Example: $\int \sin^3(x)\cos^2(x)dx = \int \sin^2(x)\cos^2(x)\sin(x)dx$

$= \int \left(1-\cos^2(x)\right)\cos^2(x)\sin(x)dx$

*make a substitution:* let $u = \cos(x)$; then $-du = \sin(x)\,dx$.

$= \int \sin^3(x)\cos^2(x)dx = -\int (1-u^2)u^2 du = \int \left(u^4 - u^2\right)du$

*You know how to proceed from here.*

Note: Use technology to check your results, but proceed carefully when comparing answers. You may find the **expand** key to be useful.

1. $\int \cos^3(x)dx$

2. $\int \sin^4(x)\cos(x)dx$

3. $\int \sin^6(x)\cos^3(x)dx$

4. $\int \dfrac{\sin^3(x)}{\cos^4(x)}dx$

5. $\int \sec^3(x)\tan^3(x)dx$

6. $\int \tan^2(x)dx$

7. $\int \tan^4(x)\sec^4(x)dx$

8.2(2) Integrating trigonometric functions with only even powers of the sine and cosine:

In these we make liberal use of the identities

(1) $\cos(2x) = 2\cos^2(x) - 1$ or

(2) $\cos(2x) = 1 - 2\sin^2(x)$

Example: $\int \sin^2(x)dx$

In order to integrate this function we substitute for $\sin^2(x)$ by solving equation (2) for $\sin^2(x)$ and then substitute to obtain

$$\int \sin^2(x)dx = \int \frac{1-\cos(2x)}{2}dx = \frac{1}{2}\int (1-\cos(2x))\,dx = \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right) + C$$

Note: Use technology to check your results, but proceed carefully when comparing answers. You may find the **expand** and **tCollect** *(under the trig heading on the algebra menu)* keys to be useful.

Try these:

1. $\int \cos^2(x)dx$

2. $\int \sin^2(x)\cos^2(x)dx$

3. $\int \sin^2(x)\cos^4(x)dx$

4. $\int 8\cos^4(2\pi x)dx$

5. $\int \sec^4(x)dx$