

AP Calculus BC

Lesson 8.4 Partial Fractions.

8.4(1) Consider the indefinite integral: $\int \frac{6x+7}{(x+2)^2} dx$

- a) Find an answer using technology
- b) Look at the answer and construct an integral expression that would give that answer.
- c) Compare the integrand in part b) to the integrand in the original problem. What do you know about them?
- d) Find constants A and B such that $\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$.

Try again by using the **expand** key on the algebra menu.

8.4(2) Do each of the following:

- a) Find constants A and B such that $\frac{x-1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$.

Try again by using the **expand** key on the algebra menu.

Now evaluate $\int \frac{x-1}{(x-2)(x+1)} dx$

Check using technology. Are the solutions equivalent? (Think log rules...)

- b) Rewrite $\frac{4x+2}{x-1}$ in the form $A + \frac{B}{x-1}$ where A and B are constants.
Hint: Use long division.

Try again using either **expand** or **propFrac**.

Evaluate $\int \frac{4x+2}{x-1} dx$ and check using technology.

- c) Rewrite $\frac{x^2}{1+x^2}$ in the form $A + \frac{B}{1+x^2}$, where A and B are constants.

Hint: Use long division.

Try again using either **expand** or **propFrac**.

Evaluate $\int \frac{x^2}{1+x^2} dx$ and check using technology

- d) Use long division to write $\frac{(x-2)^2}{x+2}$ in the form $Ax + B + \frac{C}{x+2}$.

Try again using either **expand** or **propFrac**.

Evaluate $\int \frac{(x-2)^2}{x+2} dx$ and check using technology

8.4(3) Find the indicated antiderivative using two methods.

a) $\int \frac{dx}{x^2 - 4}$

b) $\int \frac{dx}{x^2 + 3x}$

c) $\int \frac{dx}{x^2(x+1)^2}$

d) $\int \frac{x^3}{x^2 + 1} dx$

e) $\int \frac{x^2 dx}{x^2 + x - 6}$

f) $\int \frac{x^4 + 2x}{x^2 + 1} dx$

8.4(4) Find constants A, B, C, and D so that $\frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$

Find $\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} dx$

8.4(5) Find constants A, B, C, and D so that $\frac{x^3 - 2x}{(x^2 + 2x + 2)^2} = \frac{Ax + B}{x^2 + 2x + 2} + \frac{Cx + D}{(x^2 + 2x + 2)^2}$

Find $\int \frac{x^3 - 2x}{(x^2 + 2x + 2)^2} dx$

8.4(6) Find constants a and b such that $x^4 + 1 = (x^2 + ax + 1)(x^2 + bx + 1)$

Find $\int \frac{x^2 + 1}{x^4 + 1} dx$

8.4(7) Find each antiderivative:

a) $\int \frac{dx}{x^3 + x}$

b) $\int \frac{x}{(x+1)(x^2+1)} dx$

c) $\int \frac{x^2 + 2}{(x^2 + 1)^2} dx$

d) $\int \frac{3x + 1}{(x^2 + 2x + 5)^2} dx$